SCTC Analysis Estimates Low-Frequency -3-dB Point

Use Short-Circuit Time Constants To Guide The Choice Of Bypass-And Coupling-Capacitor Values In Linear-Circuit Design.

Open-circuit time constants is a method for approximating the high-frequency -3-dB point \( \omega_H \) that was detailed in a previous article (Electronic Design Special Analog Issue, June 24, 1993, p. 41). An analogous short-circuit time constants (SCTC) procedure allows designers to approximately calculate the low frequency -3-dB point \( \omega_L \) of a linear circuit using the circuit's low-frequency incremental model.

The complete model for a typical ac-coupled transistor amplifier has four capacitors in it, two of which are due to internal transistor capacitances (Fig. 1a). In many instances, the transistor amplifier has a wide mid-frequency range where gain is constant (Fig. 1b). Therefore, the amplifier's full incremental model can be split into three simpler models—low frequency, mid-band, and high frequency.

In the low-frequency amplifier model, the effects of bypass and coupling capacitors dominate, and internal transistor capacitances behave as open-circuits. The mid-band model can ignore the effects of all circuit capacitances. In the high-frequency amplifier model, internal transistor capacitances dominate, and all bypass and coupling capacitors behave as short-circuits.

It's simple to determine the form of the transfer function for the amplifier.

1. A typical ac-coupled transistor amplifier (a) has a wide mid-frequency range in which gain is constant (b). Consequently, the full incremental model can be split into three simpler models.
in Figure 1a if only low- and mid-frequency behavior is considered. In addition, the analysis won’t account for high-frequency behavior due to internal transistor capacitances. The circuit has two capacitors, so there are two independent poles in the transfer function. Also, there’s a zero at zero frequency due to the coupling capacitor $C_C$. Finally, there’s a zero at higher frequency due to the emitter bypass capacitor $C_E$. A transfer function that meets these requirements is:

$$v_{out} = \frac{Ks}{(t_1 s + 1)(t_2 s + 1)}$$

In this transfer function results in the gain-versus-frequency plot that’s shown in Figure 1b for low and mid frequencies. Although it won’t be proven here, all zeros of the transfer function are at lower frequencies than the highest pole. Therefore, the transfer function may be approximated by assuming that the zeros are at zero frequency:

$$v_{out} = \frac{Ks^2}{(t_1 + 1)(t_2 + 1)}$$

Multiplying out the denominator results in:

$$v_{out} = \frac{Ks^2}{t_1 t_2 s^2 + (t_1 + t_2) s + 1}$$

Generally, each coupling and bypass capacitor has a relatively large value. This results in each time constant $\tau$ being relatively big. Therefore, throw out the +1 term in the denominator of the equation to make the following approximation valid at low frequencies:

$$v_{out} = \frac{Ks}{t_1 t_2 s + (t_1 + t_2)}$$

Using standard Bode-plot methods, it is clear that the –3-dB point of this transfer function occurs when:

$$\omega_L = \frac{t_1 + t_2}{t_1 t_2} = \frac{1}{\frac{1}{t_1} + \frac{1}{t_2}}$$

Equation 5 approximates the low-frequency breakpoint as the summation of two easily calculated frequencies. Following a line of reasoning similar to that used in open-circuit time constants analysis, the low-frequency breakpoint of a network is approximated by:

$$\omega_L \approx \sum_{j} \frac{1}{\tau_j}$$

where each $\tau_j$ is an RC time constant of every individual bypass or coupling capacitor with all other bypass and decoupling capacitors in the circuit shorted. Even though this example works through the math for a circuit with two capacitors, the method may be with equal validity to networks with more bypass and coupling capacitors.

**DESIGN EXAMPLE**

Consider the design of a single-transistor amplifier (Fig. 2). The circuit uses a 2N3904 transistor, and has the following design specifications: a source resistance $R_s$ of 1 kΩ, gain of greater than 100, and a low-frequency –3-dB point $f_L$ of less than 1 kHz.

The SCTC analysis considers only the low-frequency behavior of the amplifier. Bias-point and load-resistor values have been chosen to give a mid-band gain of approximately –100. SCTC will help select reasonable values for the capacitors $C_C$ and $C_E$ based on the specification of low-frequency –3-dB point.

The transistor has the following parameters: collector current, $I_C = 2$ mA; $r_e = 160 \quad \Omega$; $r_s = 2.6 \quad k\Omega$; $g_m = 0.08$ mhos; and $h_{fe} = 200$ (Fig. 3a). The two short-circuit time constants for $C_C$ and $C_E$ are to be called $\tau_1$ and $\tau_2$. SCTC show that to meet the bandwidth specification, the following must hold:

$$1 + \frac{1}{\tau_1} + \frac{1}{\tau_2} < 2\pi \times 1000$$

To find $\tau_1$, the short-circuit time constant for $C_C$, first turn off the input source $v_{in}$ by shorting it. Then, short circuit $C_E$ and find the resistance $R_{in}$ across the $C_E$ terminals (Fig. 3b). The time constant $\tau_1 = R_{in} \cdot C_C$. Because a brief inspection reveals that the $g_m v_{ps}$ generator has no effect on $R_{in}$, the resistance is:

$$R_{1s} = R_s + (R_{81} + R_{82}) (r_s + r_x) = 3.5 \quad k\Omega$$

Find $\tau_2$, the short-circuit time constant for $C_E$, by short circuiting $C_C$ and calculating the resistance $R_{2s}$ across the $C_E$ terminals (Fig. 3c). For the sake of simplicity, ignore the $R_{81}$ and $R_{82}$ resistances, because they are much larger than $R_s$. The resistance $R_{2s}$ is found by applying a test-voltage source at the emitter terminal of the transistor and measuring the resulting current:

$$I_T = \frac{E_T}{R_o + r_x}$$

The first term in Equation 6 is current through the emitter resistor due to the voltage source; the second term is current through $R_{81}$ and $r_x$ due to the voltage source alone; and the third term is current due to the $g_m v_{ps}$ generator. Recognizing that $g_m v_{ps} = h_{fe}$, the resistance facing $C_C$ is given by:

$$R_{2s} = R_s + \frac{h_{fe}}{1 + h_{fe}} = 18 \quad \Omega$$

2. THIS AMPLIFIER is built with just one 2N3904 transistor, and has gain of greater than 100 for $\omega_L$ of less than 1 kHz.
Now, the final choice of capacitor values is constrained by Equation 7, which sets the minimum values of time constants $\tau_2$ and $\tau_3$. Arbitrarily “divide up” the time constants equally between the two capacitors.

\[
\frac{1}{\tau_1} = \frac{1}{R_{1s} C_C} < 2\pi \times 500
\]

(11)

\[
\frac{1}{\tau_2} = \frac{1}{R_{2s} C_E} < 2\pi \times 500
\]

(12)

This gives design values:

\[
C_C > \frac{1}{2\pi \times 500 \times R_{1s}} > 0.09 \mu F
\]

(13)

\[
C_E > \frac{1}{2\pi \times 500 \times R_{2s}} > 18 \mu F
\]

(14)

Chosen off-the-shelf values are $C_C = 0.1 \mu F$, and $C_E = 20 \mu F$.

The method of short-circuit time constants shows that a relatively small value of coupling capacitor $C_c$ will suffice due to the large resistance seen at its terminals. It also makes sense that $C_E$ must be a large capacitor value, because the output resistance of an emitter follower is very small.

**CAVEAT EMPTOR**

It’s obvious by inspecting Figure 2 that capacitors $C_C$ and $C_E$ affect the low- and mid-frequency behavior of the circuit. For instance, if capacitor $C_C$ were open-circuited, the mid-band gain of the amplifier would be zero. If capacitor $C_E$ were open-circuited, the mid-band gain of the amplifier would be about –2.

There are cases, however, where bypass capacitors have little or no effect on the mid-frequency performance of an amplifier. For instance, in a common-emitter amplifier with a cascode, the resistive network biasing the base of the cascode transistor is usually bypassed with a capacitor. The bypassing ensures better high-frequency performance, but has little effect at low and mid frequencies.

This capacitor should not be included in the short-circuit time constants analysis. So, the moral is: think carefully about how the circuit operates before applying the short-circuit time constants method to a given capacitor in a circuit.

**RESULTS**

A Spice run shows that the gain of the circuit is approximately –110, with a low-frequency –3-dB point of 500 Hz. (Note that Spice was not used for design, only for verification). The answer obtained from using SCTC was not exact, but who cares? It’s more important that engineers develop design insight by using such methods.

Short-circuit time constants offer guidance for choosing bypass- and coupling-capacitor values. Using this method, a design may be optimized for preferred capacitor values based on such criteria as size or cost. For instance, this article’s design example divided up the time constant equally between the two capacitors, but any other combination will also work. As with the method of open-circuit time constants, all results are approximate. But the value of such methods is in the design insight the method provides to the designers, not exact answers.

**References:**


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