

Electromagnetic and Electromechanical Engineering Principles Notes 01 Basics

Marc T. Thompson, Ph.D.
Thompson Consulting, Inc.

9 Jacob Gates Road
Harvard, MA 01451

Phone: (978) 456-7722

Email: marctt@thompsonrd.com

Website: <http://www.thompsonrd.com>

© Marc Thompson, 2006-2008

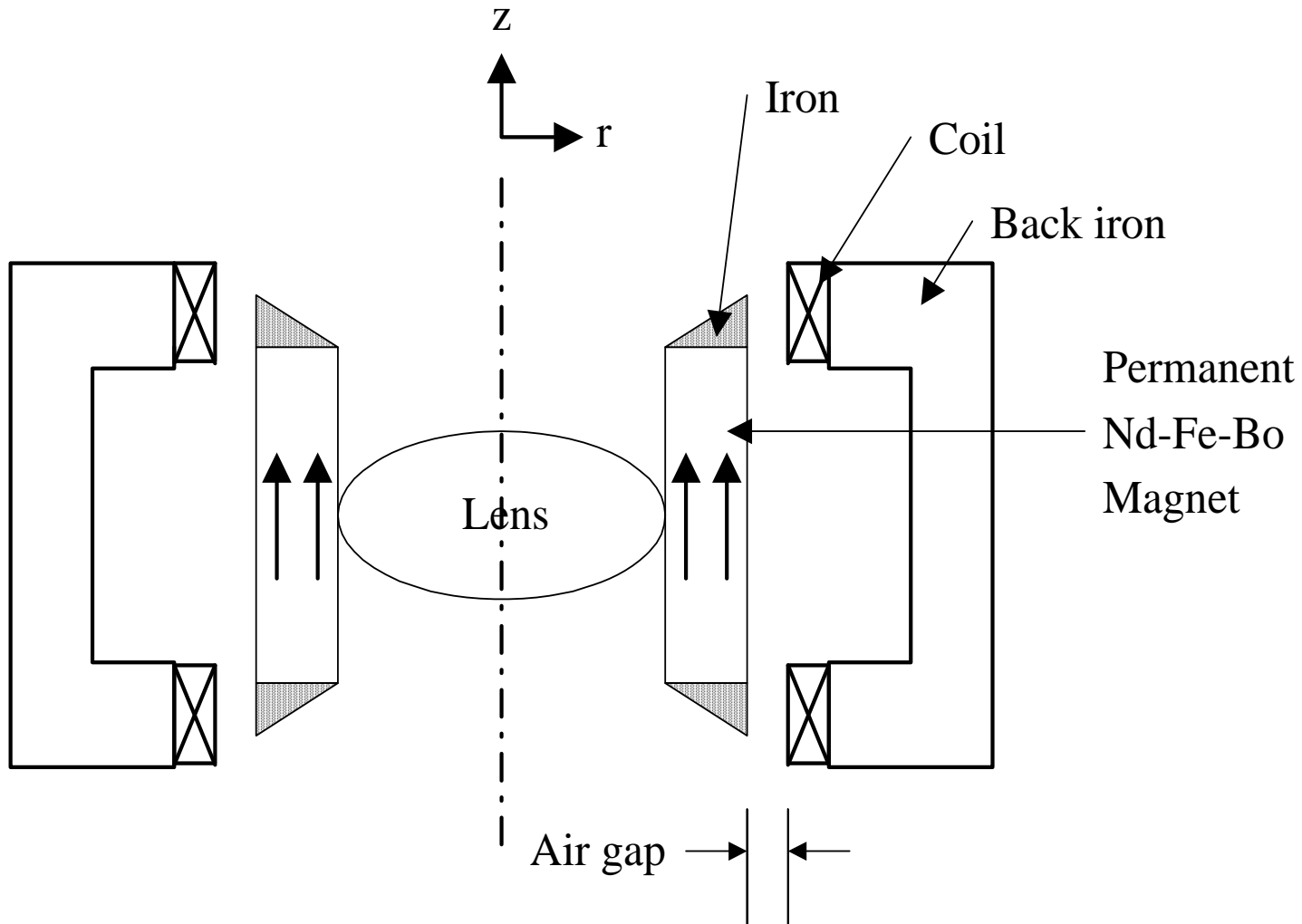
Portions of these notes reprinted from Fitzgerald, Electric Motors, 6th edition

Background of Instructor

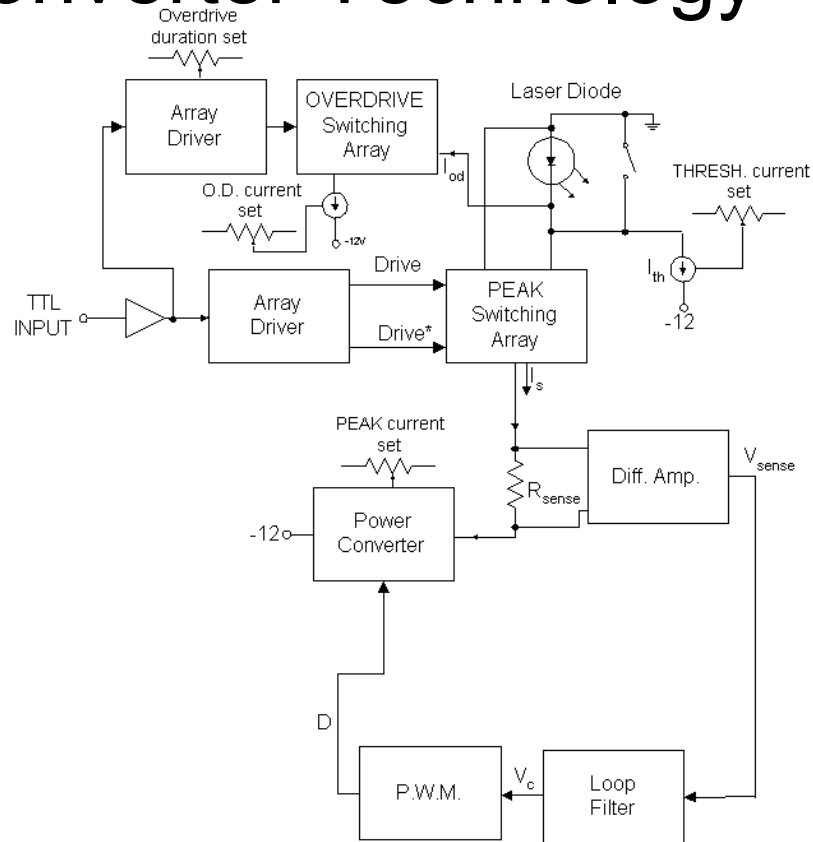
- BSEE ('85), MS ('92), and PhD ('97) from MIT
- Doctoral work done in the Laboratory for Electromagnetic and Electronic Systems (L.E.E.S.) at MIT in electrodynamic Maglev
- Consultant in analog, magnetics, power electronics, electromagnetics, magnetic braking and failure analysis
- Adjunct Professor at Worcester Polytechnic Institute, Worcester MA teaching graduate courses in analog design, power electronics, electromechanics and power distribution
- Have also taught for University of Wisconsin since 2006

Lens Actuator

- For high speed laser printing



High Power Laser Diode Driver Based on Power Converter Technology

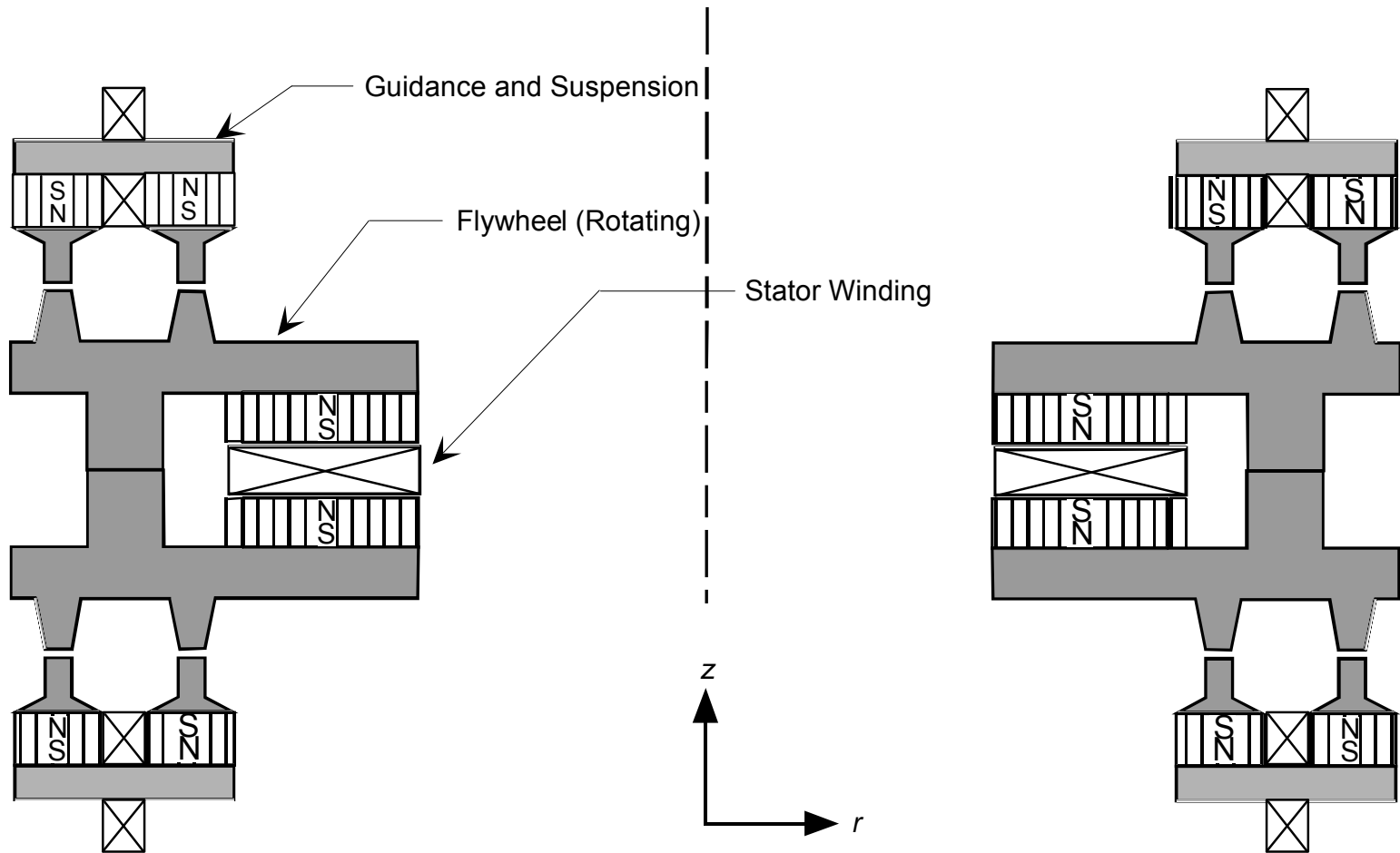


See:

1. B. Santarelli and M. Thompson, U.S. Patent #5,123,023, "Laser Driver with Plural Feedback Loops," issued June 16, 1992
2. M. Thompson, U.S. Patent #5,444,728, "Laser Driver Circuit," issued August 22, 1995
3. W. T. Plummer, M. Thompson, D. S. Goodman and P. P. Clark, U.S. Patent #6,061,372, "Two-Level Semiconductor Laser Driver," issued May 9, 2000
4. Marc T. Thompson and Martin F. Schlecht, "Laser Diode Driver Based on Power Converter Technology," *IEEE Transactions on Power Electronics*, vol. 12, no. 1, Jan. 1997, pp. 46-52

Magnetically-Levitated Flywheel Energy Storage System

- For NASA; $P = 100\text{W}$, energy storage = 100 W-hrs



Transrapid Maglev

- Currently in operation from Pudong (Shanghai), connecting airport to subway line
- Operational speed is 431 km/hr

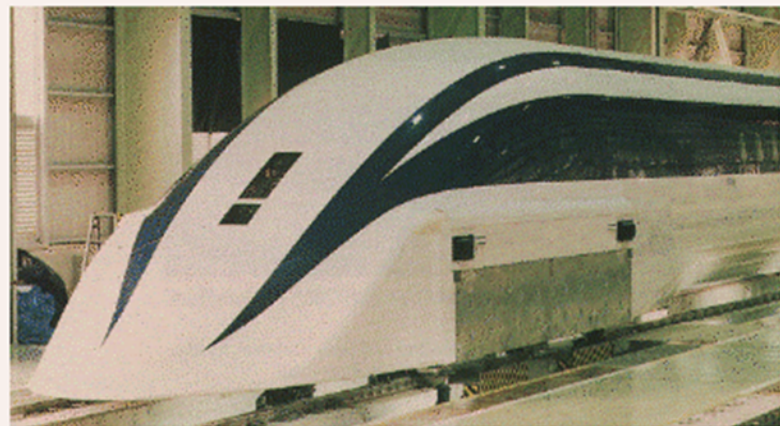


Japanese EDS Maglev

- Slated to run parallel to the high-speed Shinkansen line
- On December 2, 2003, this three-car train set attained a maximum speed of 581 km/h in a manned vehicle run



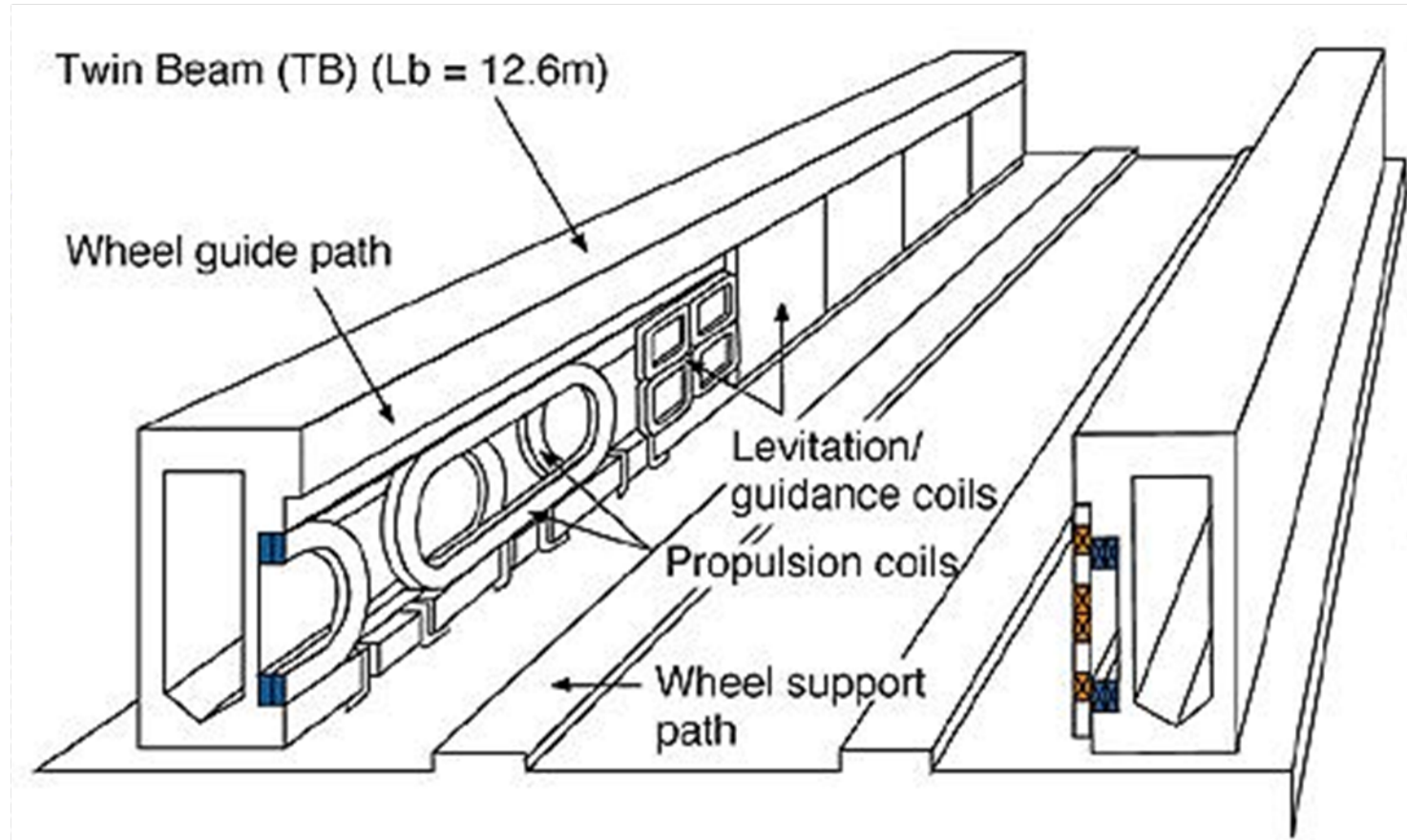
Aero-wedge Style



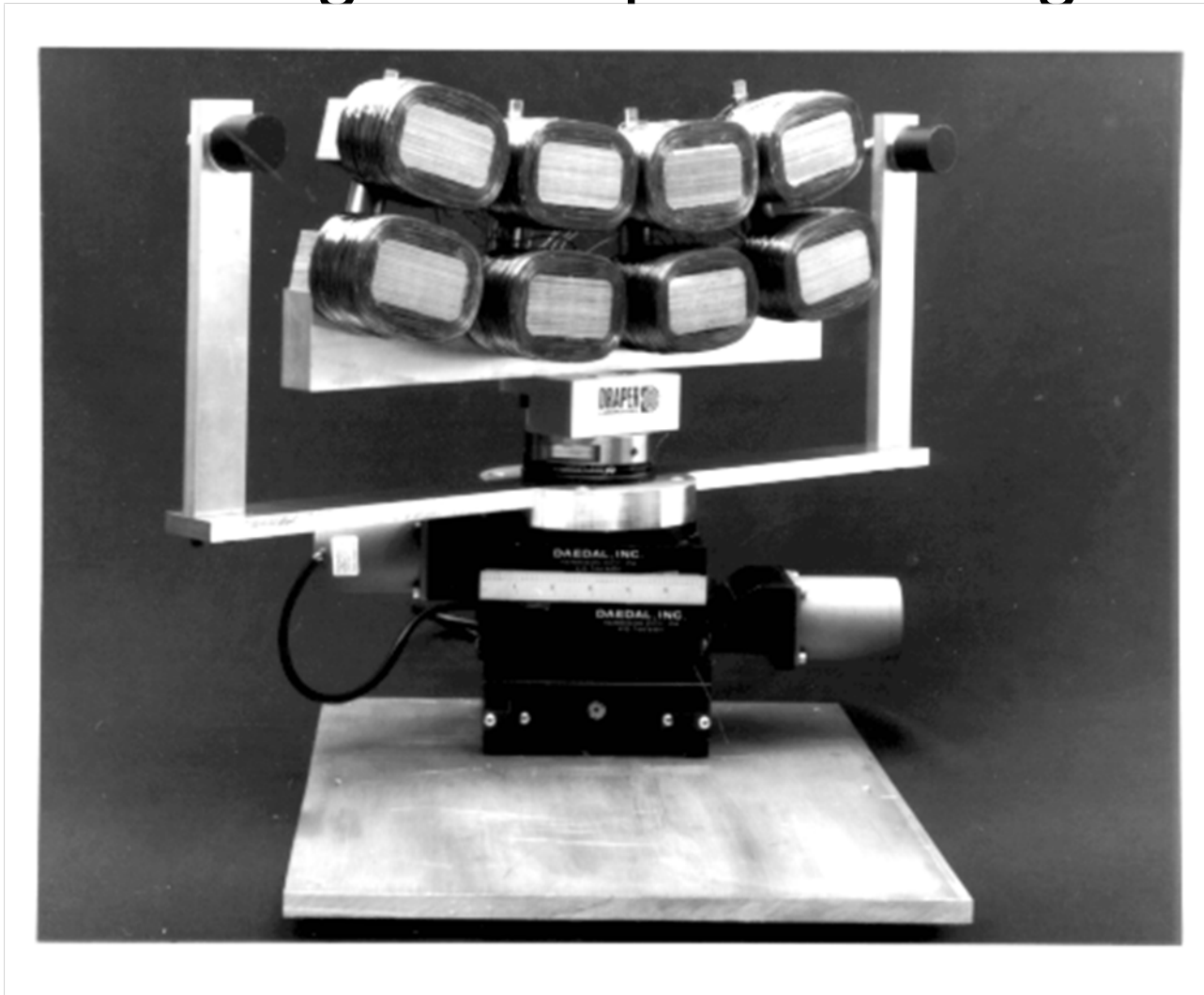
Double Cusp Style

Japanese EDS Guideway

- This view shows levitation, guidance and propulsion coils

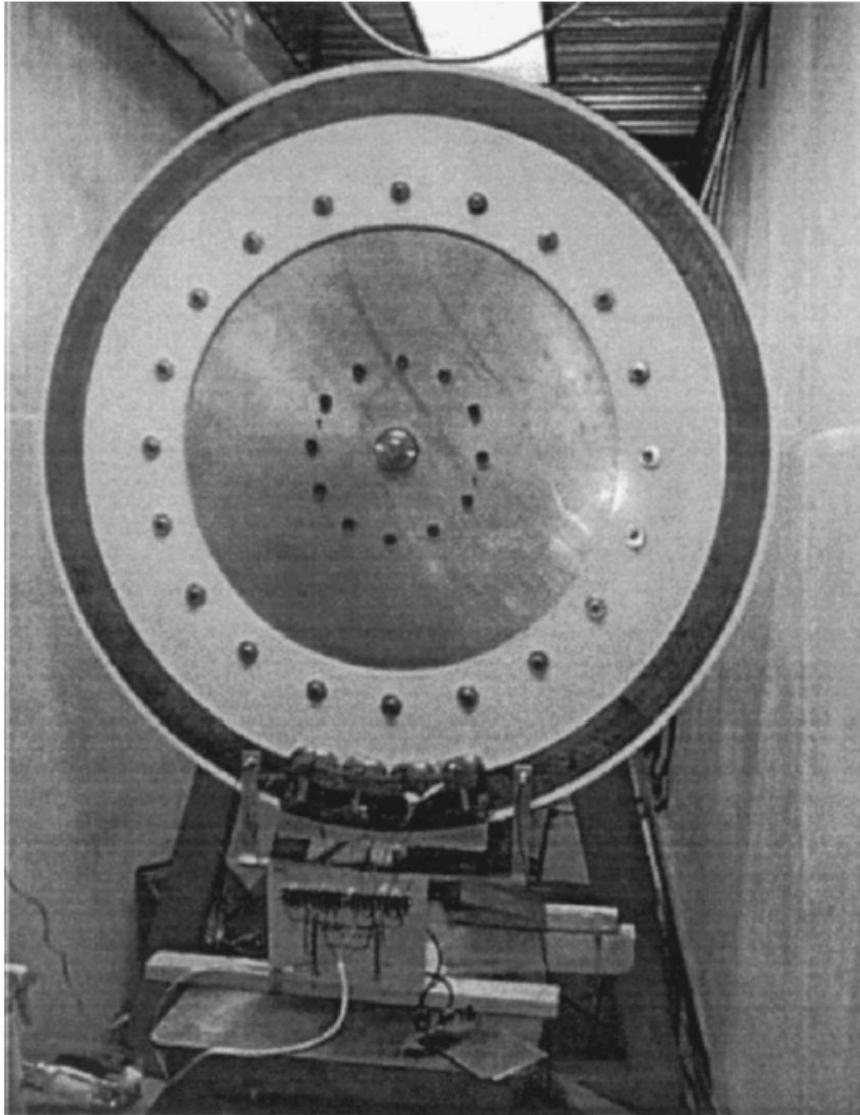


MIT Maglev Suspension Magnet

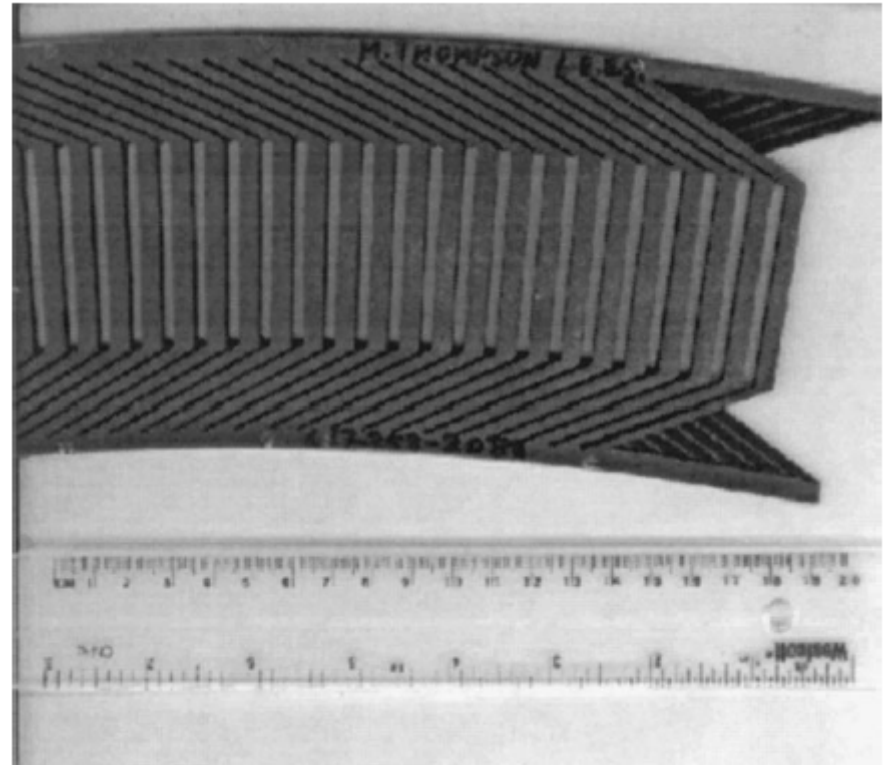


M. T. Thompson, R. D. Thornton and A. Kondoleon, "Flux-canceling electrodynamic maglev suspension: Part I. Test fixture design and modeling," *IEEE Transactions on Magnetics*, vol. 35, no. 3, May 1999 pp. 1956-1963

MIT Maglev Test Fixture



Electromechanics

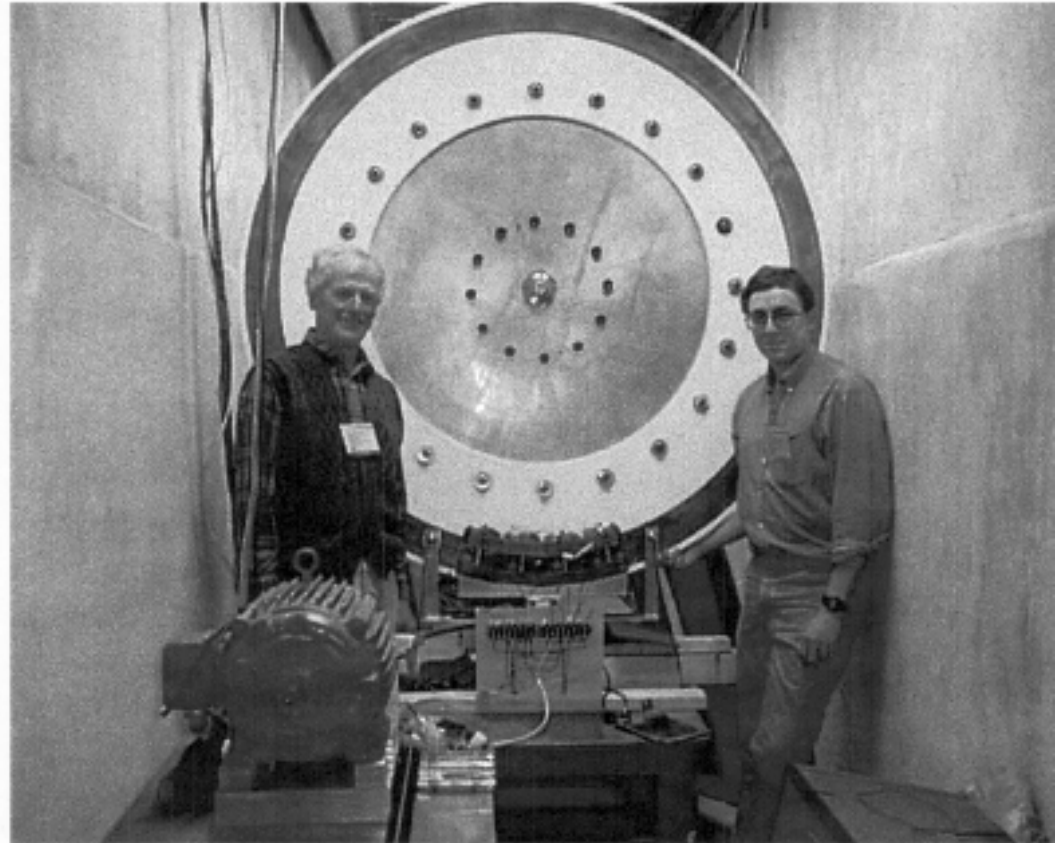


M. T. Thompson, R. D. Thornton and A. Kondoleon, "Flux-cancelling electrodynamic maglev suspension: Part I. Test fixture design and modeling," *IEEE Transactions on Magnetics*, vol. 35, no. 3, May 1999 pp. 1956-1963

Basics

MIT EDS Maglev Test Facility

- 2 meter diameter test wheel
- Max. speed 1000 RPM (84 m/s)
- For testing “flux canceling” HTSC Maglev
- Sidewall levitation



References:

1. Marc T. Thompson, Richard D. Thornton and Anthony Kondoleon, “Scale Model Flux-Canceling EDS Maglev Suspension --- Part I: Design and Modeling,” *IEEE Transactions on Magnetics*, vol. 35, no. 3, May 1999, pp. 1956-1963
2. Marc T. Thompson and Richard D. Thornton, “Scale Model Flux-Canceling EDS Maglev Suspension --- Part II: Test Results and Scaling Laws,” *IEEE Transactions on Magnetics*, vol. 35, no. 3, May 1999, pp. 1964-1975

Permanent Magnet Brakes

- For roller coasters
- Braking force $> 10,000$ Newtons per meter of brake



Reference: <http://www.magnetarcorp.com>

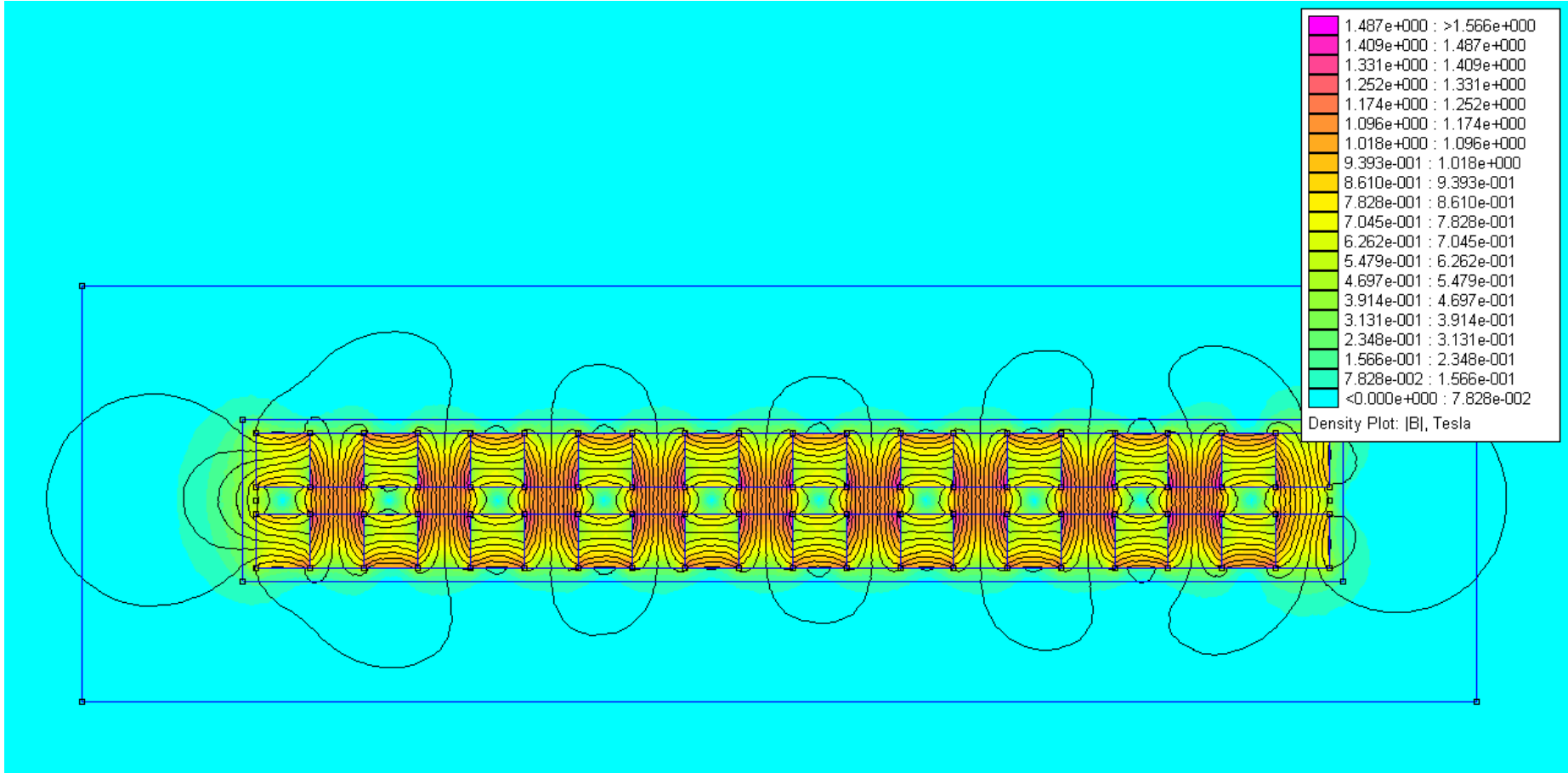
Halbach Permanent Magnet Array

- Special PM arrangement allows strong side (bottom) and weak side (top) fields
- Applicable to magnetic suspensions (Maglev), linear motors, and induction brakes



Halbach Permanent Magnet Array

- 2D FEA modeling



Photovoltaics

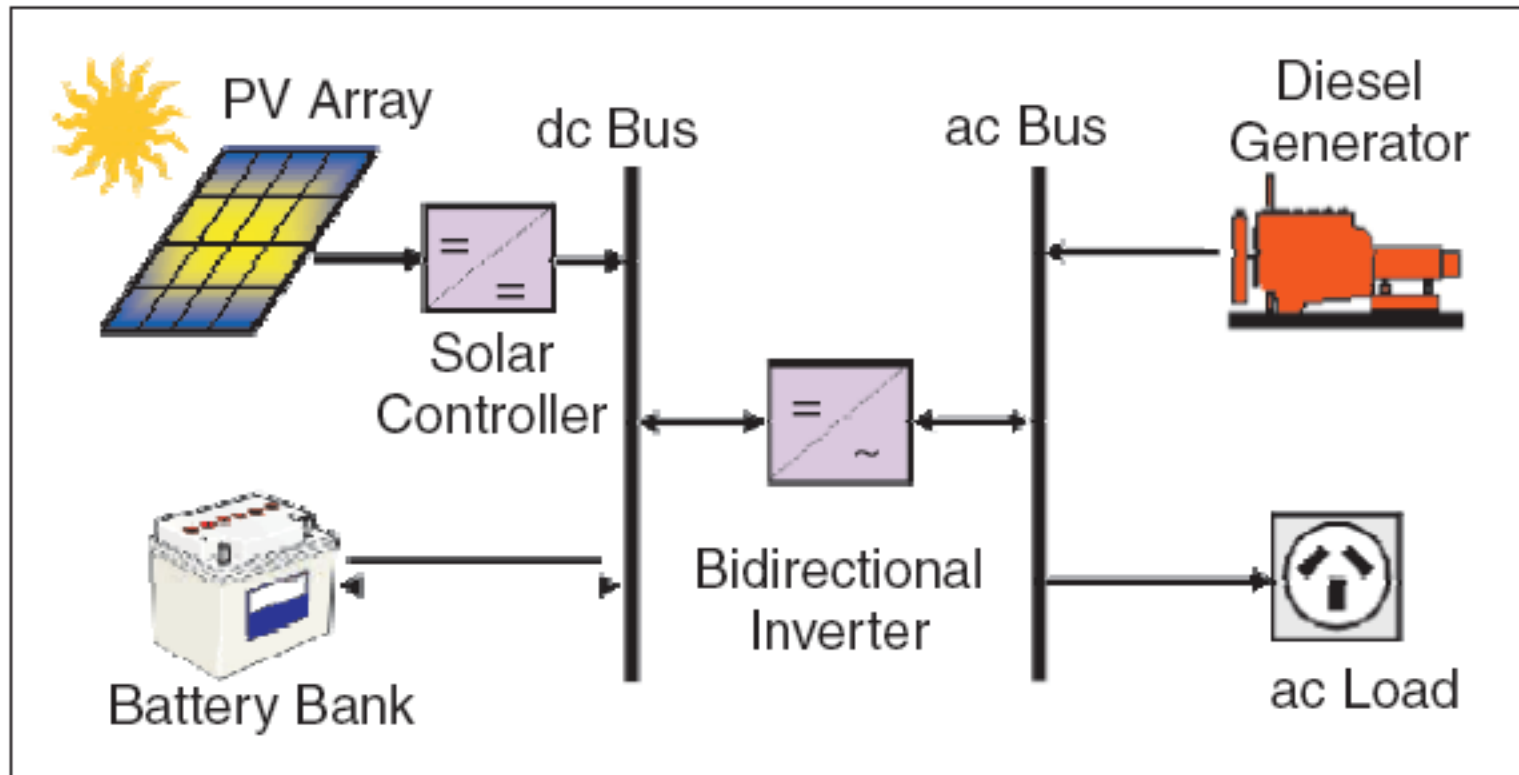
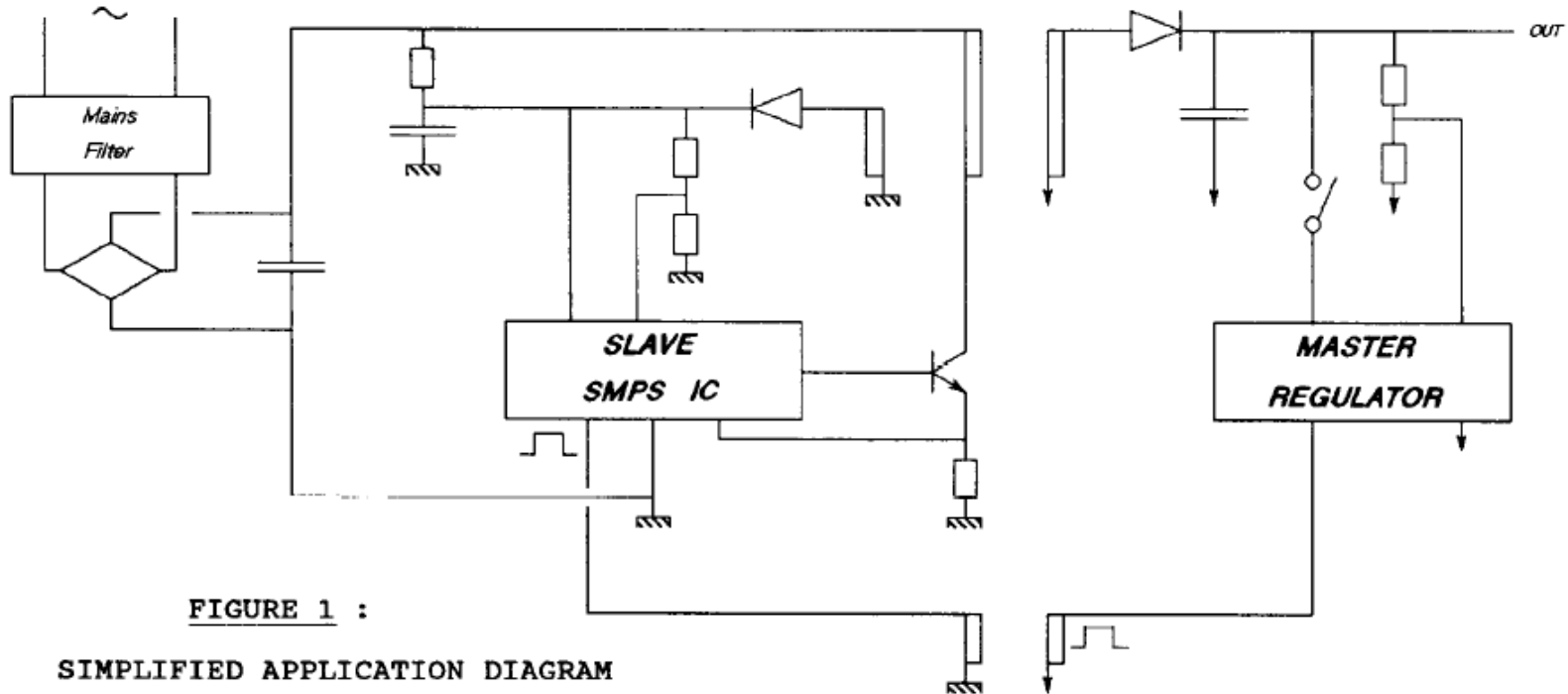


Fig. 1. Typical hybrid system layout.

S. Druyea, S. Islam and W. Lawrance, "A battery management system for stand-alone photovoltaic energy systems," *IEEE Industry Applications Magazine*, vol. 7, no. 3, May-June 2001, pp. 67-72

Offline Flyback Power Supply



P. Maige, "A universal power supply integrated circuit for TV and monitor applications," *IEEE Transactions on Consumer Electronics*, vol. 36, no. 1, Feb. 1990, pp. 10-17

Transcutaneous Energy Transmission

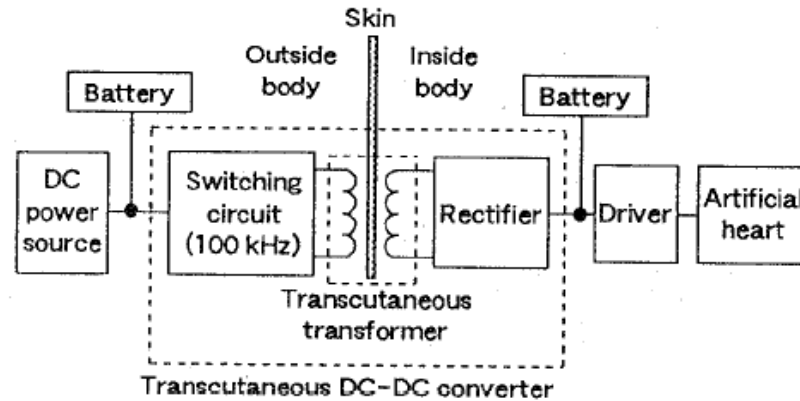


Fig. 1. Transcutaneous energy transmission system for an implantable artificial heart.

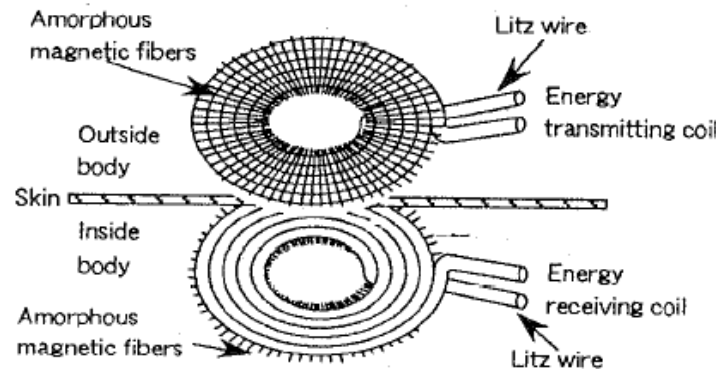
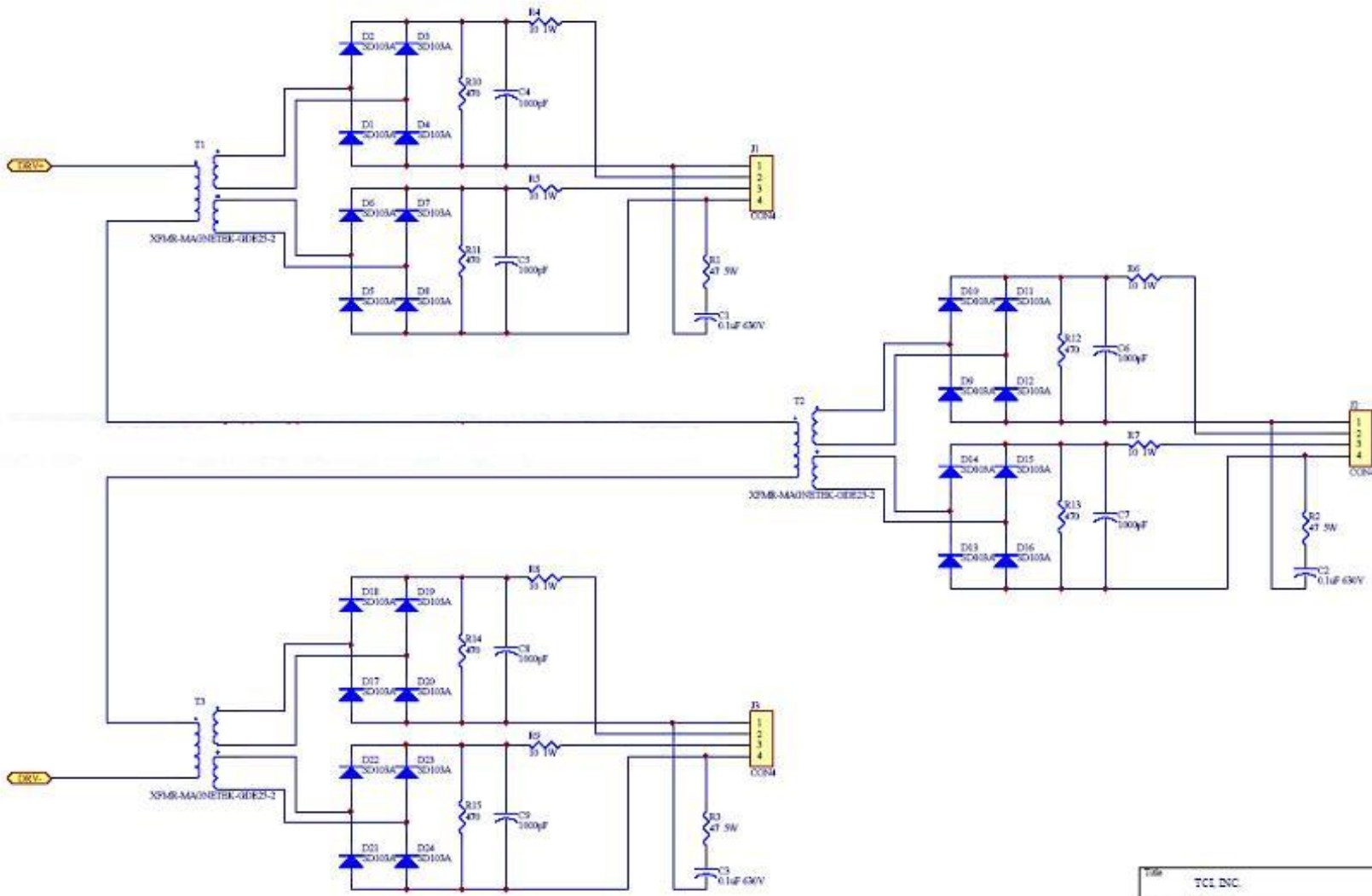


Fig. 2. Transcutaneous transformer.

Reference: H. Matsuki, Y. Yamakata, N. Chubachi, S.-I. Nitta and H. Hashimoto, "Transcutaneous DC-DC converter for totally implantable artificial heart using synchronous rectifier," *IEEE Transactions on Magnetics*, vol. 32, no. 5, Sept. 1996, pp. 5118 - 5120

50 KW Inverter Switch



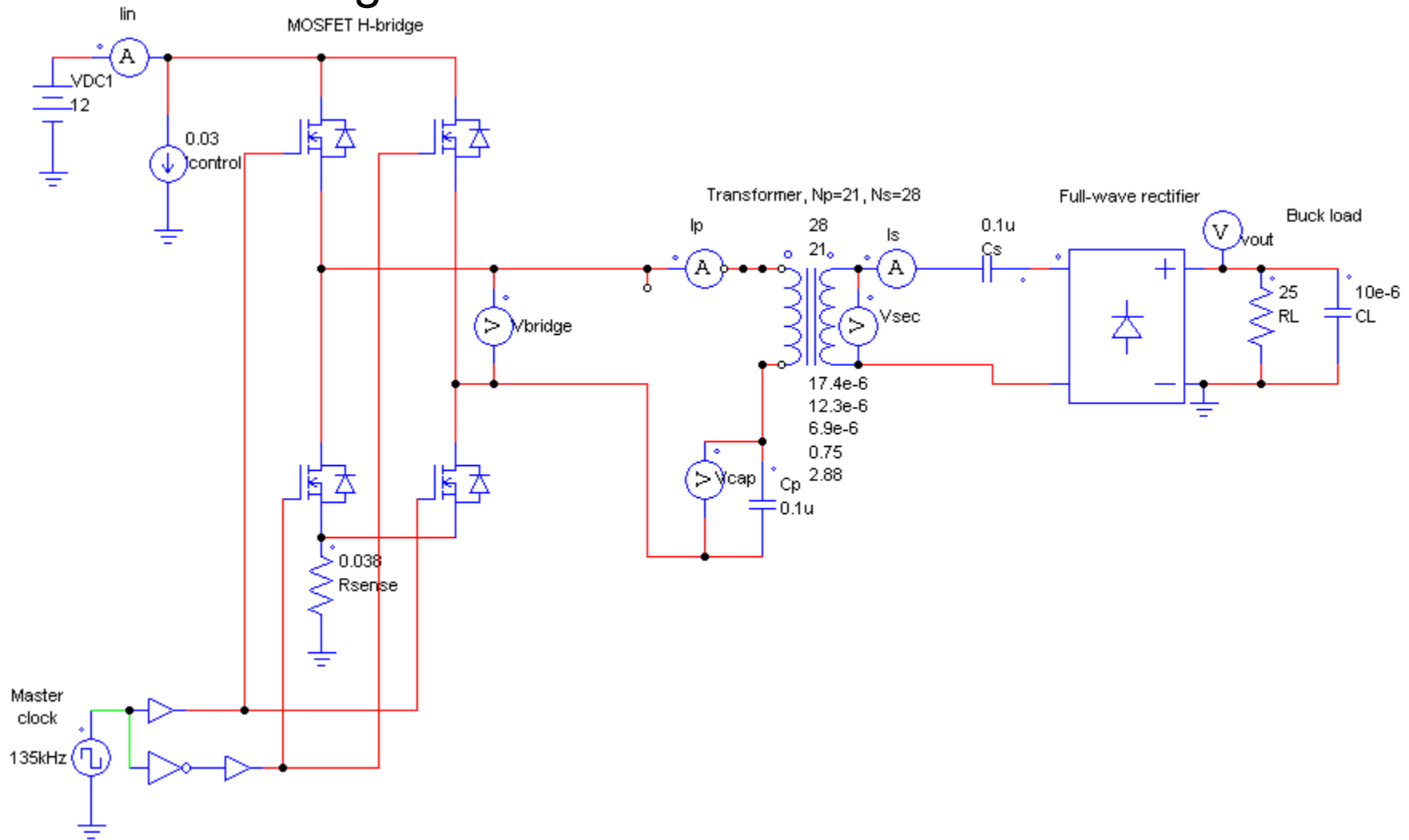
Title	FCL INC.	
Rev	Number	Revision
B	INVERTER SWITCH	A

Transformer Failure Analysis

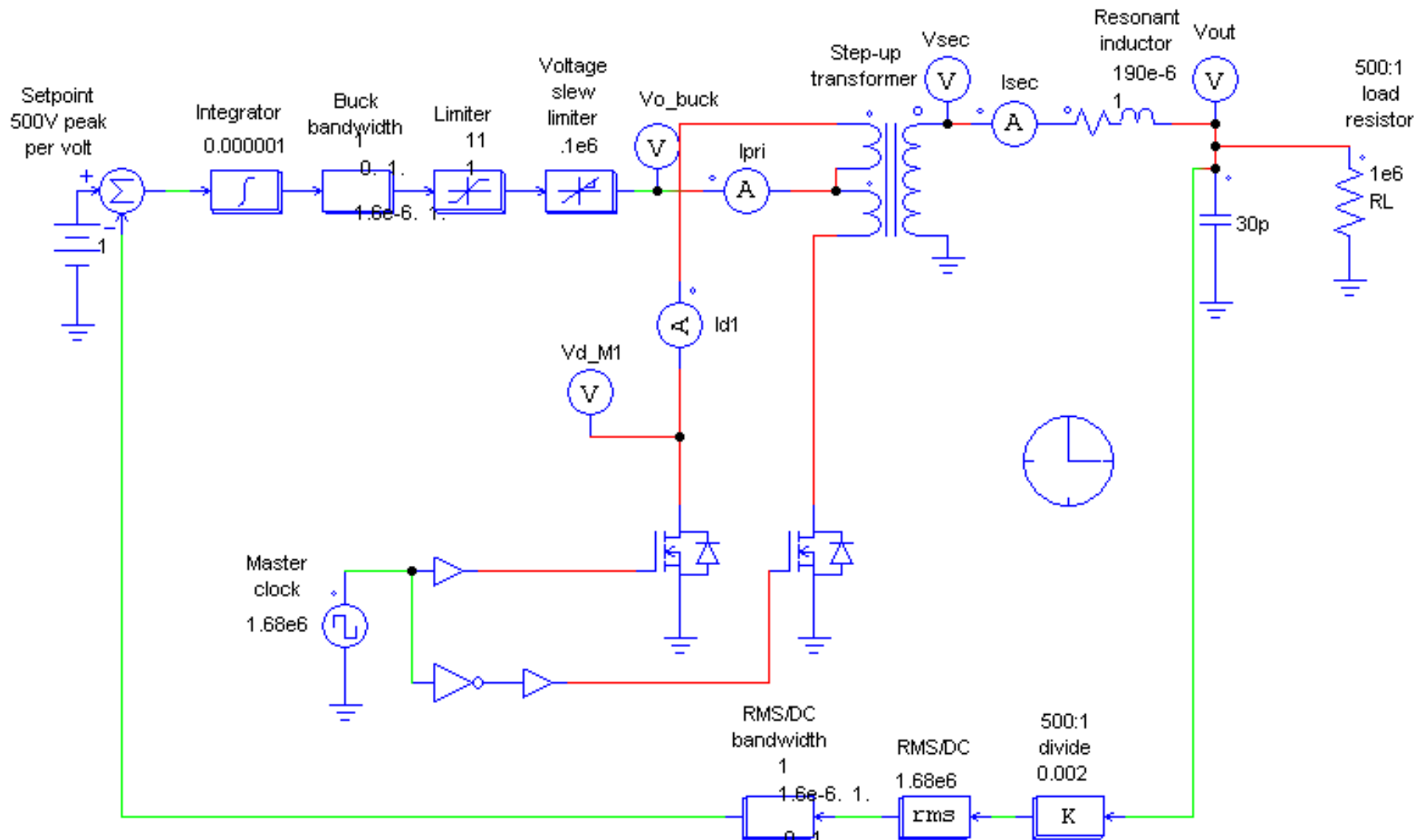


Non-Contact Battery Charger

- Modeled using PSIM

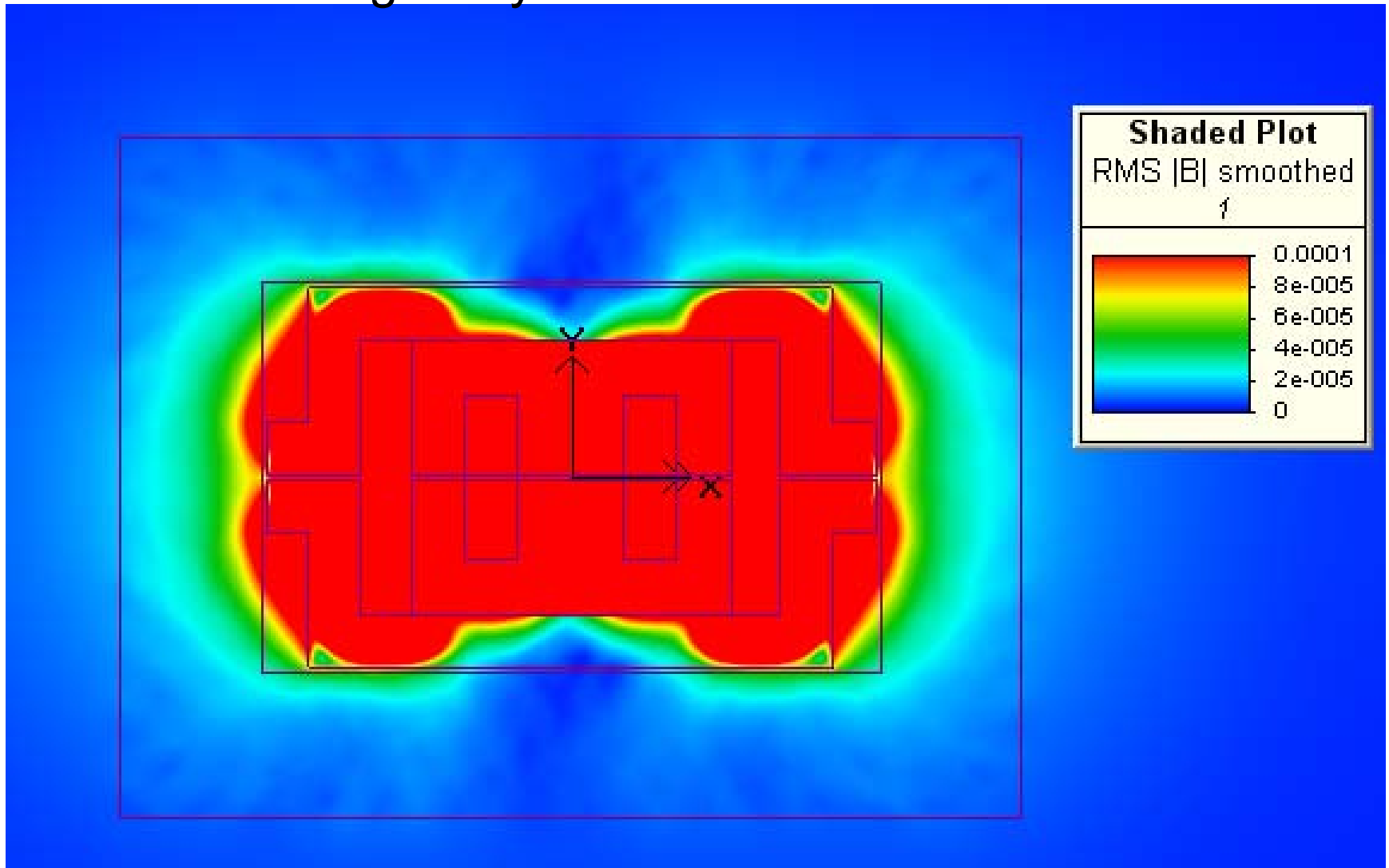


High Voltage RF Supply



60 Hz Transformer Shielding Study

- Modeled using Infolytica 2D and 3D



Course Overview

- Day 1 --- Basics of power, 3 phase power, harmonics, etc.
 - Day 2 --- Magnetics, transformers and energy conversion
 - Day 3 --- Basic machines
-
- Numerous examples are done throughout the 3 days, some from Fitzgerald, Kingsley and Umans

Course Overview --- Day 1

Day	Material Covered	Hours	Slides
1	Basics		
	- Attendee sign-in.....	8:00-8:30	
	- Introductions, instructor background, and course 3-day overview.....	8:30-8:45	1-26
	- Basic circuit analysis concepts.....	8:45-9:15	27-58
	- Basic single-phase power, complex number review, 3-phase, RMS.....	9:15-10:15	59-96
	- Morning break.....	10:15-10:30	
	- Power: real, imaginary and instantaneous; P, Q, S and power factor.....	10:30-11:00	97-104
	- Power cables; AWG and circular mils; cable impedance.....	11:00-11:45	105-126
	- Single line drawings.....	11:45-12:00	127-130
	- Lunch.....	12:00-1:00	
	- Power factor and power factor correction issues.....	1:00-1:30	131-151
	- Harmonics, Fourier series; total harmonic distortion (THD).....	1:30-2:00	152-174
	- Begin 3-phase; 6-pulse rectifiers; 12-pulse rectifiers.....	2:00-2:45	175-188
	- Afternoon break.....	2:45-3:00	
- 3-phase circuits and 3-phase power; neutral currents with and without harmonics; nonlinear loads.....	3:00-3:45	189-201	
- Summarize.....	3:45-4:00	202	

Note: we'll start at 7:30 on days #2 and #3

Supplemental notes: Inductance calculation methods (if time permits)

Supplemental notes: Maglev (if time permits)

Course Overview --- Day 2

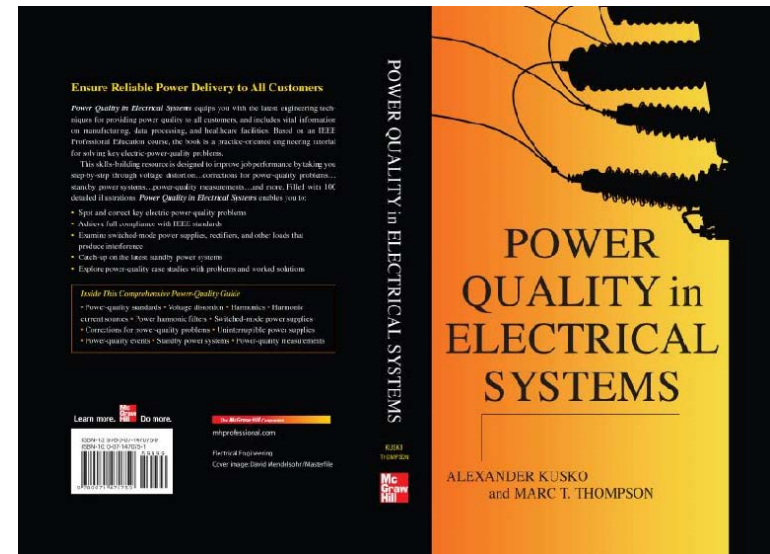
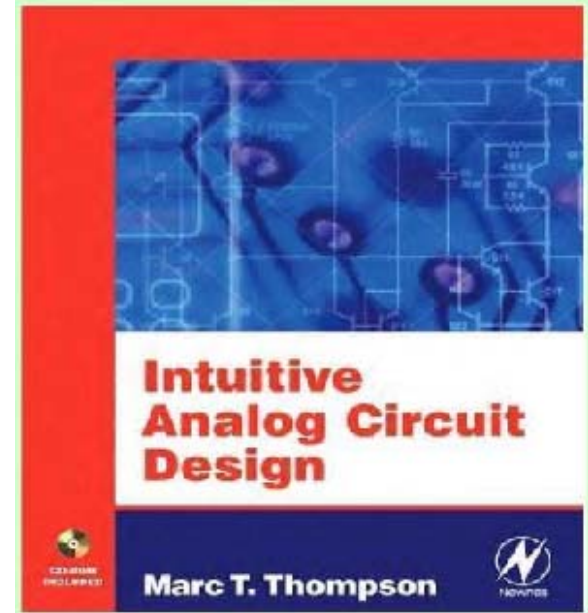
2	<p><u>Magnetics and energy conversion</u></p> <ul style="list-style-type: none"> - Review of Maxwell's equations; demo; concepts of magnetic fields; Ampere's, Faraday's and Gauss' magnetic laws; Lorentz force law; magnetic circuits..... - Morning break..... - Soft magnetic materials (steel); hysteresis and core losses..... - Hard magnetic (permanent magnets) and PM circuits..... - Lunch - Comments on superconductors; basic transformers: analysis, equivalent circuits..... - Afternoon break..... - Per-unit system..... - Begin electromechanical conversion; begin forces and torques..... - Summarize..... 	<p>7:30-9:30</p> <p>9:30-9:45</p> <p>9:45-10:45</p> <p>10:45-12:00</p> <p>12:00-1:00</p> <p>1:00-2:45</p> <p>2:45-3:00</p> <p>3:00-3:15</p> <p>3:15-4:00</p> <p>4:00</p>	<p>1-54</p> <p></p> <p>55-76</p> <p>77-139</p> <p></p> <p>140-208</p> <p></p> <p>209-216</p> <p>217-240</p> <p>241</p>
---	---	--	--

Course Overview --- Day 3

3	<u>Basic machines</u>		
	- Finish forces and torques with demo and 2 examples.....	7:30-8:30	1-24
	- Introduction to AC and DC machines.....	8:30-9:30	25-41
	- Morning break.....	9:30-9:45	
	- MMF of windings; rotating magnetic fields in rotating machinery; generated voltage.....	9:45-11:00	42-76
	- Synchronous machines.....	11:00-12:00	77-97
	- Lunch.....	12:00-1:00	
	- Linear and PM synchronous machines.....	1:00-1:30	98-108
	- Induction machines.....	1:30-2:45	109-133
	- Afternoon break.....	2:45-3:00	
	- Finish induction machines.....	3:00-3:30	134-149
	- Induction (eddy current) brakes.....	3:30-4:00	150-178
	- Summarize.....	4:00-4:15	179, various

Basic Circuit Analysis Concepts

- Some background in circuits to get us all on the same page
 - First-order and second-order systems
 - Resonant circuits, damping ratio and Q
- Reference for this material: M. T. Thompson, *Intuitive Analog Circuit Design*, Elsevier, 2006 and Kusko/Thompson *Power Quality in Electrical Systems*, McGraw-Hill, 2007



First-Order Systems

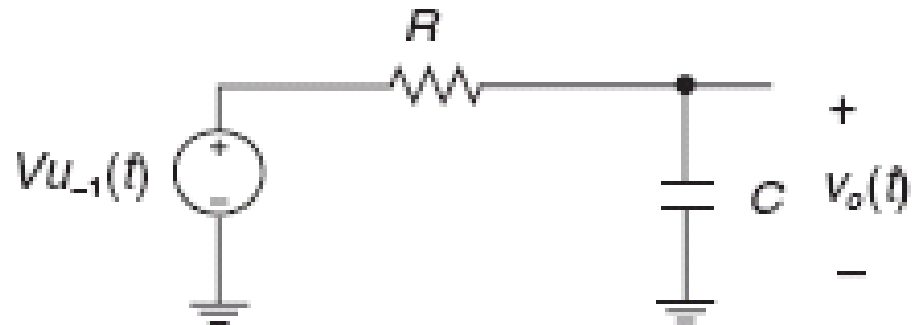
- A system with a single energy-storage element is a “first-order” system
- Step voltage driven RC lowpass filter

$$v_o(t) = V(1 - e^{-\frac{t}{\tau}})$$

$$i_r(t) = \frac{V}{R} e^{-\frac{t}{\tau}}$$

$$\tau = RC$$

$$\tau_R = 2.2\tau$$



$$\omega_h = \frac{1}{\tau}$$

$$f_h = \frac{\omega_h}{2\pi}$$

$$\tau_R = \frac{0.35}{f_h}$$

First-Order Systems --- Some Details

- Frequency response:

$$H(s) = \frac{1}{\tau s + 1}$$

- Phase response:

$$\angle H(s) = -\tan^{-1}(\omega\tau)$$

- -3 dB bandwidth:

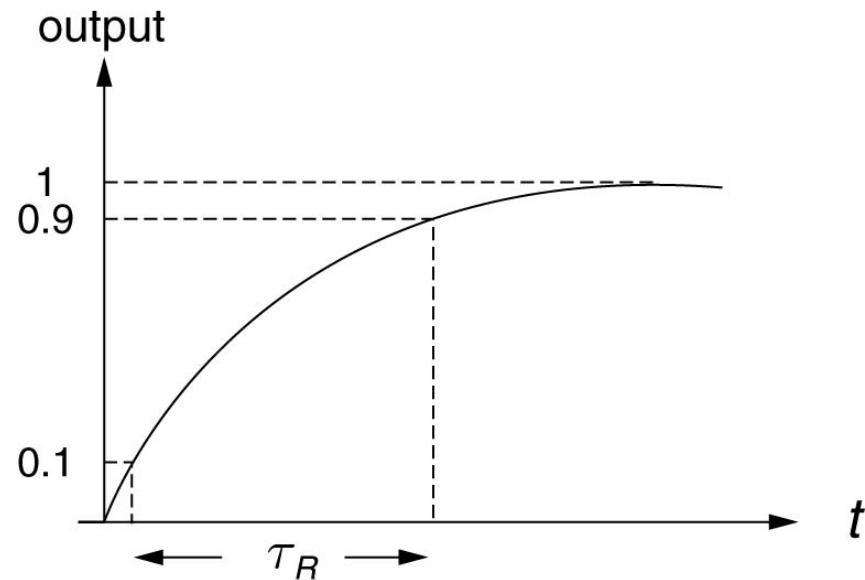
$$\omega_h = \frac{1}{\tau}$$

$$f_h = \frac{\omega_h}{2\pi}$$

Step Response 10 - 90% Risetime

- Often-measured figure of merit for systems
- Defined as the time it takes a step response to transition from 10% of final value to 90% of final value
- This plot is for a first-order system with no overshoot or ringing

$$\tau_R = 2.2\tau$$



Relationship Between Risetime and Bandwidth

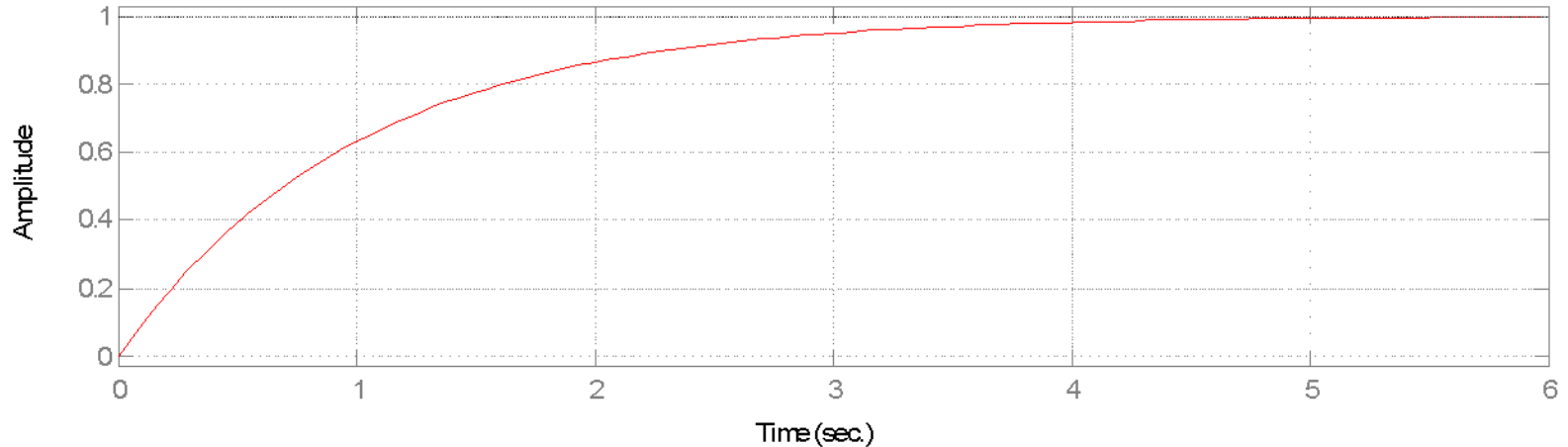
- Exact for a first-order system (with one pole):

$$\tau_R = \frac{0.35}{f_h}$$

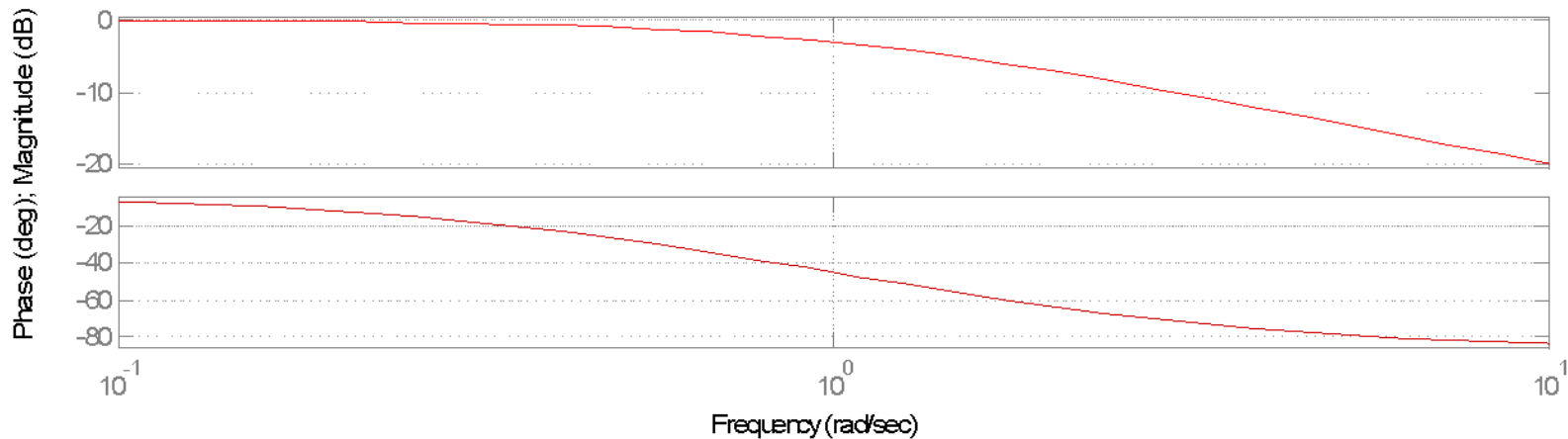
- Approximate for higher-order systems

First-Order Step and Frequency Response

- Single pole at -1 rad/sec. Step Response

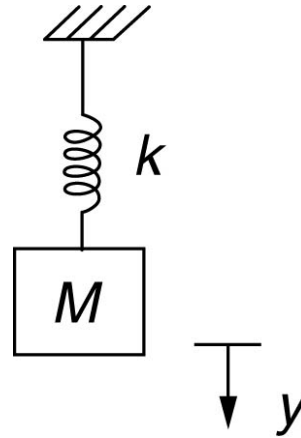


Bode Diagrams



Second-Order Mechanical System

- 2 energy storage modes in mass and spring



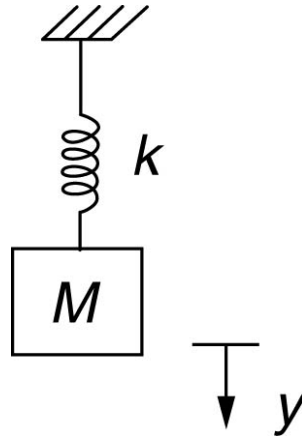
Spring force: $f_y = -ky$

Newton's law for moving mass: $f_y = -ky = M \frac{d^2y}{dt^2}$

Differential equation for mass motion: $M \frac{d^2y}{dt^2} + ky = 0$

Guess a solution of the form: $y(t) = Y_o \sin(\omega t)$

Second-Order Mechanical System



$$y(t) = Y_o \sin(\omega t)$$

Put this proposed solution into the differential equation:

$$M(-\omega^2 Y_o \sin(\omega t)) + k(Y_o \sin(\omega t)) = 0$$

This solution works if: $\omega = \sqrt{\frac{k}{M}}$

Second-Order Mechanical System

- Electromechanical modeling; useful for modeling speakers, motors, acoustics, etc.

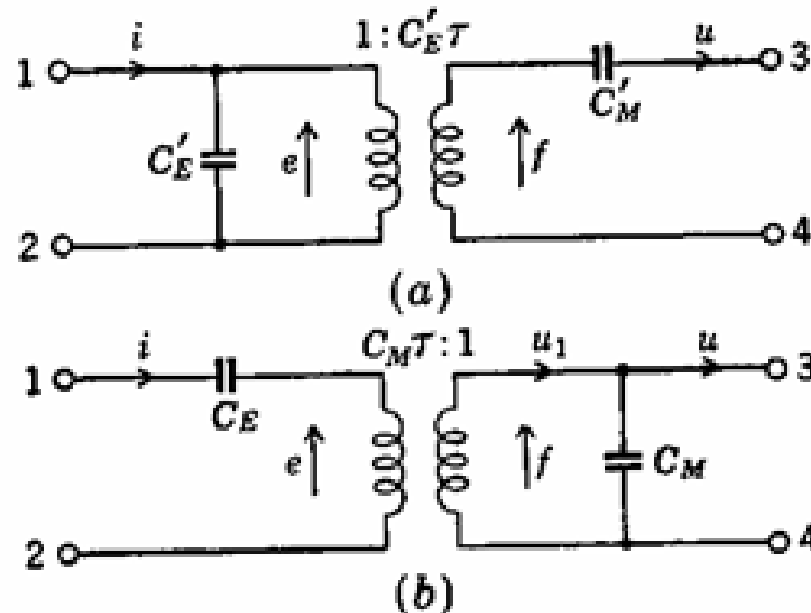
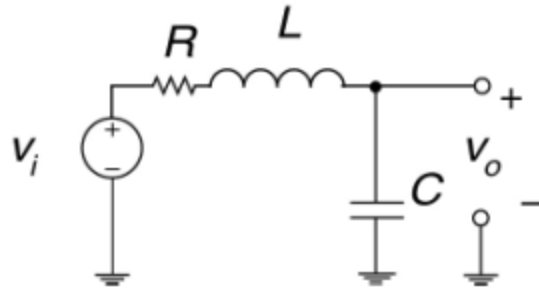


FIG. 3.37. Two forms of analogous symbols for piezoelectric transducers. The mechanical sides are of the impedance type.

Reference: Leo Beranek, *Acoustics*, Acoustical Society of America, 1954

Second-Order Electrical System



$$H(s) = \frac{v_o(s)}{v_i(s)} = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Natural frequency $\omega_n = \frac{1}{\sqrt{LC}}$

Damping ratio $\zeta = \frac{\omega_n RC}{2} = \frac{1}{2} \frac{R}{\sqrt{\frac{L}{C}}} = \frac{1}{2} \frac{R}{Z_o}$

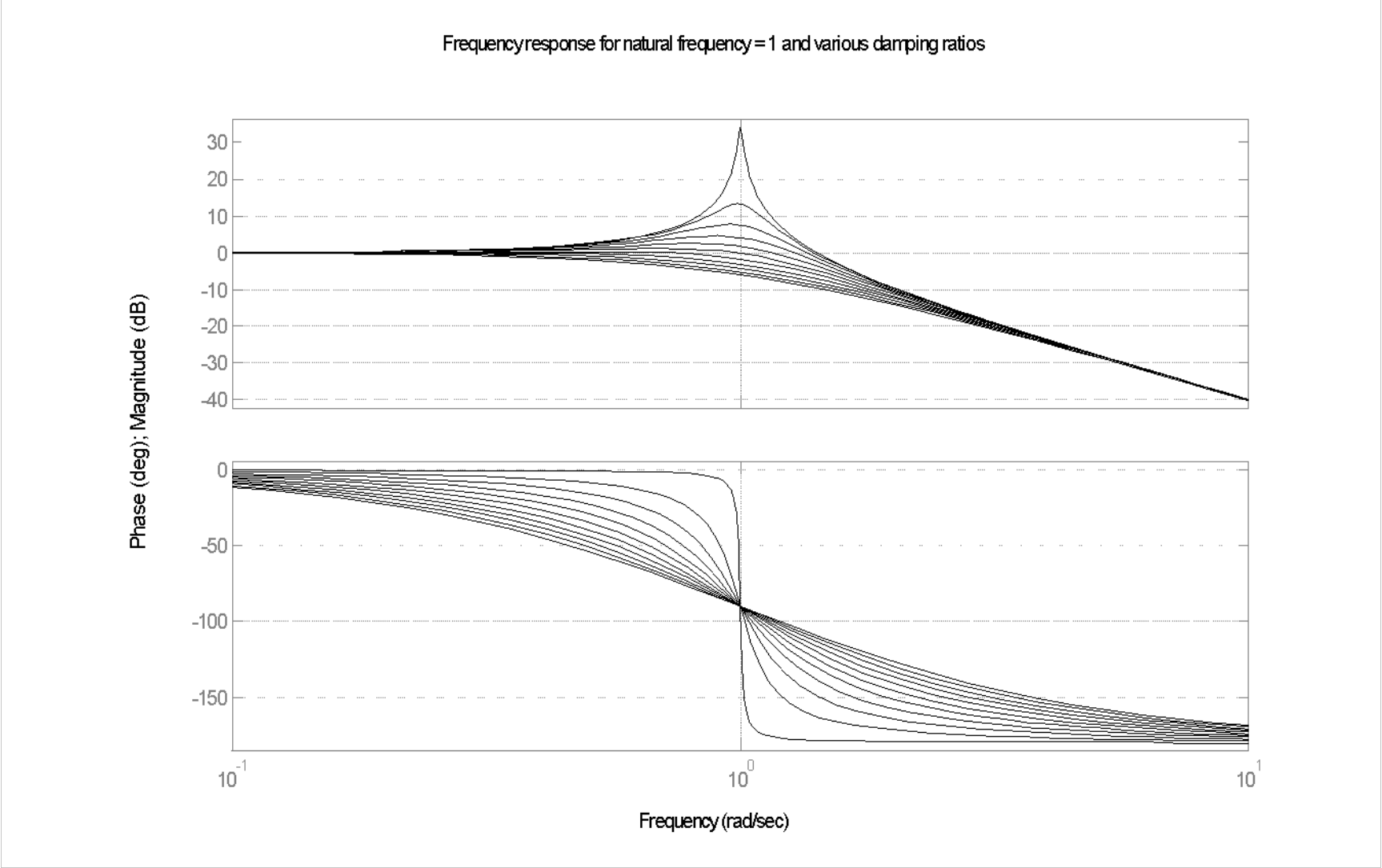
Second-Order System Frequency Response

$$H(j\omega) = \frac{1}{\frac{2j\zeta\omega}{\omega_n} + \left(1 - \frac{\omega^2}{\omega_n^2}\right)}$$

$$|H(j\omega)| = \frac{1}{\sqrt{\left(\frac{2\zeta\omega}{\omega_n}\right)^2 + \left(1 - \frac{\omega^2}{\omega_n^2}\right)^2}}$$

$$\angle H(j\omega) = -\tan^{-1} \frac{\left(\frac{2\zeta\omega}{\omega_n}\right)}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)} = -\tan^{-1} \left(\frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2}\right)$$

Second-Order System Frequency Response



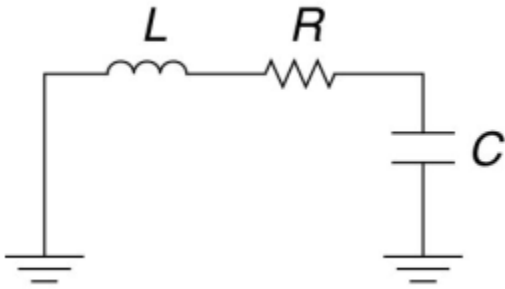
Quality Factor, or “Q”

Quality factor is defined as:

$$Q = \frac{\omega E_{\text{stored}}}{P_{\text{diss}}}$$

where E_{stored} is the peak stored energy in the system and P_{diss} is the average power dissipation.

Q of Series Resonant RLC



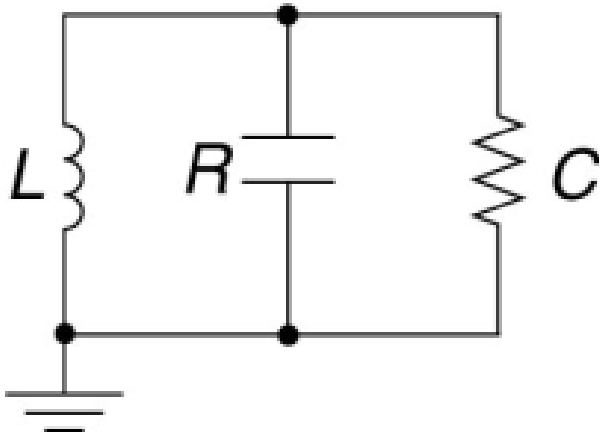
$$\omega = \frac{1}{\sqrt{LC}}$$

$$E_{stored} = \frac{1}{2} LI_{pk}^2$$

$$P_{diss} = \frac{1}{2} I_{pk}^2 R$$

$$Q = \left(\frac{1}{\sqrt{LC}} \right) \left(\frac{\frac{1}{2} LI_{pk}^2}{\frac{1}{2} I_{pk}^2 R} \right) = \frac{\sqrt{L}}{R} = \frac{Z_o}{R}$$

Q of Parallel Resonant RLC



$$\omega = \frac{1}{\sqrt{LC}}$$

$$E_{stored} = \frac{1}{2} CV_{pk}^2$$

$$P_{diss} = \frac{1}{2} \frac{V_{pk}^2}{R}$$

$$Q = \left(\frac{1}{\sqrt{LC}} \right) \left(\frac{\frac{1}{2} CV_{pk}^2}{\frac{1}{2} \frac{V_{pk}^2}{R}} \right) = \frac{R}{\sqrt{\frac{L}{C}}} = \frac{R}{Z_o}$$

Relationship Between Damping Ratio and “Quality Factor” Q

- A second order system can also be characterized by its “Quality Factor” or Q.

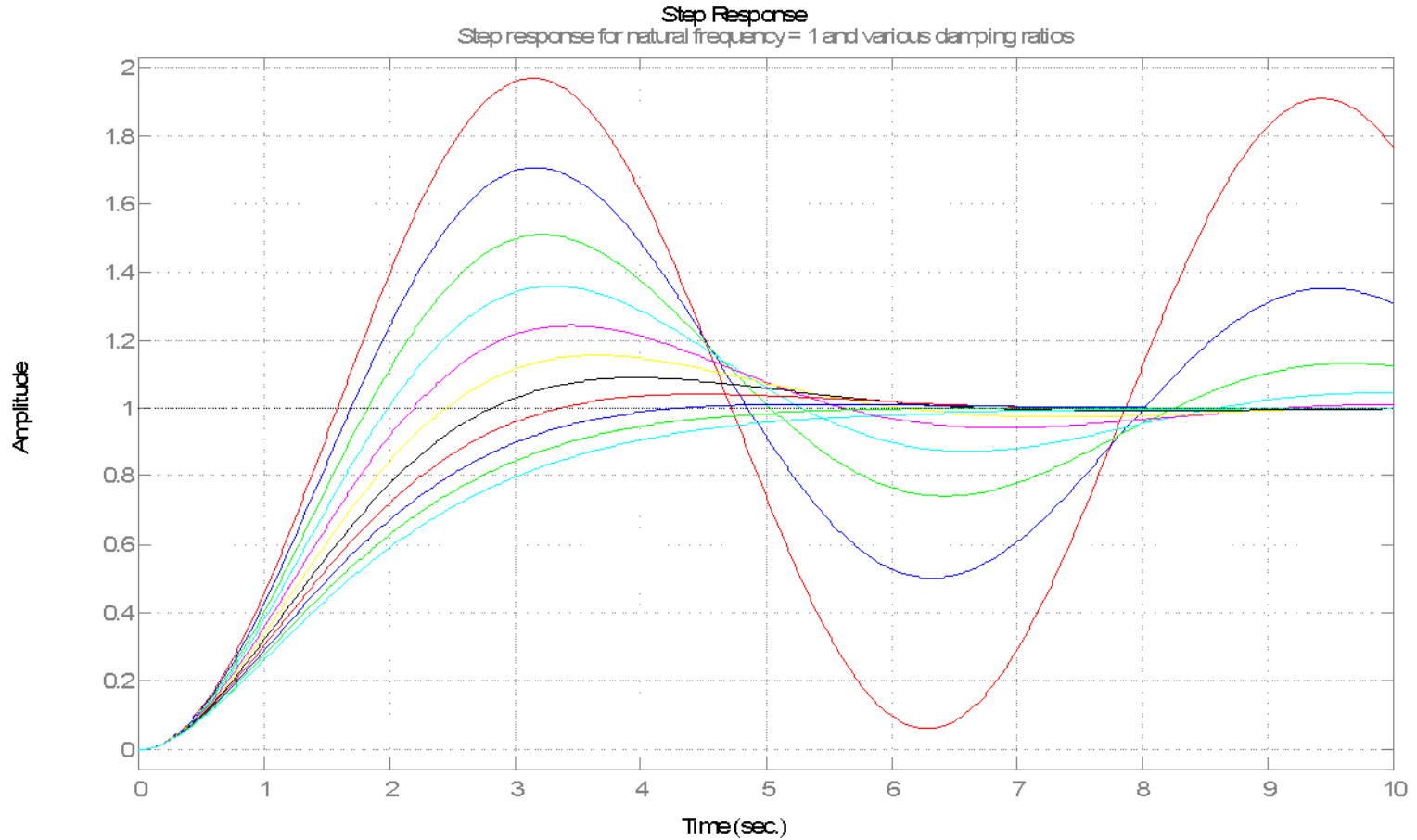
$$|H(s)|_{\omega=\omega_n} = \frac{1}{2\zeta} = Q$$

- Use Q in transfer function of series resonant circuit:

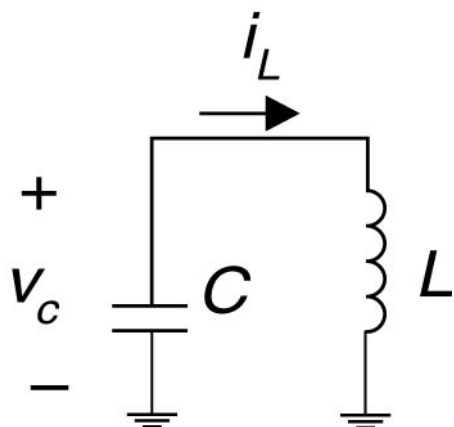
$$H(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{s}{\omega_n Q} + 1}$$

Second-Order System Step Response

- Shown for varying values of damping ratio.



Undamped Resonant Circuit



$$\frac{di_L}{dt} = \frac{v_c}{L} \quad \frac{dv_c}{dt} = \frac{-i_L}{C}$$

$$\frac{d^2v_c}{dt^2} = -\frac{1}{C} \frac{di_L}{dt} = -\frac{v_c}{LC}$$

We find the resonant frequency by guessing that the voltage $v(t)$ is sinusoidal with $v(t) = V_o \sin \omega t$. Putting this into the equation for capacitor voltage results in:

$$-\omega^2 \sin(\omega t) = -\frac{1}{LC} \sin(\omega t)$$

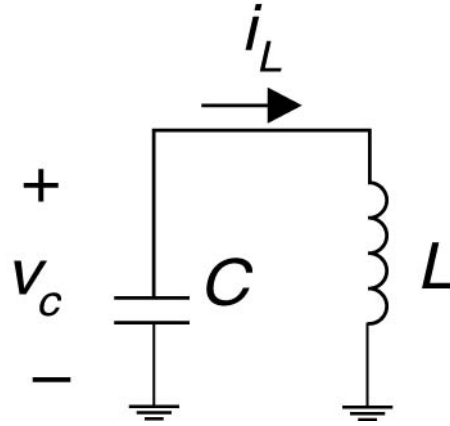
This means that the resonant frequency is the standard (as expected) resonance:

$$\omega_r^2 = \frac{1}{LC}$$

Energy Methods

Storage Mode	Relationship	Comments
Capacitor/electric field storage	$E_{elec} = \frac{1}{2} CV^2$	
Inductor/magnetic field storage	$E_{mag} = \frac{1}{2} LI^2 = \int \frac{B^2}{2\mu_o} dV$	
Kinetic energy	$E_k = \frac{1}{2} Mv^2$	
Rotary energy	$E_r = \frac{1}{2} I\omega^2$	$I \equiv$ mass moment of inertia (kg- m ²)
Spring	$E_{spring} = \frac{1}{2} kx^2$	$k \equiv$ spring constant (N/m)
Potential energy	$\Delta E_p = Mg\Delta h$	$\Delta h \equiv$ height change
Thermal energy	$\Delta E_T = C_{TH} \Delta T$	$C_{TH} \equiv$ thermal capacitance (J/K)

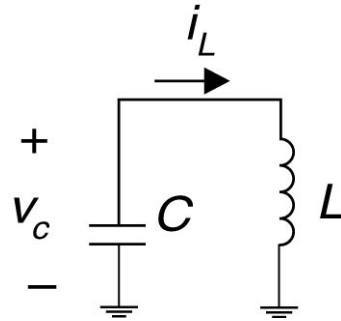
Energy Methods



By using energy methods we can find the ratio of maximum capacitor voltage to maximum inductor current. Assuming that the capacitor is initially charged to V_o volts, and remembering that capacitor stored energy $E_c = \frac{1}{2}CV^2$ and inductor stored energy is $E_L = \frac{1}{2}LI^2$, we can write the following:

$$\frac{1}{2}CV_o^2 = \frac{1}{2}LI_o^2$$

Energy Methods

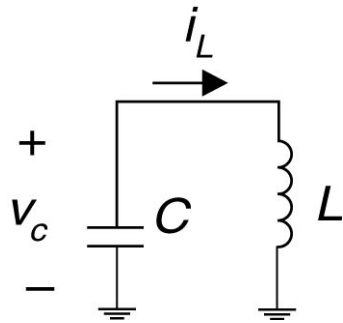


What is the inductor current? We can solve for the ratio of V_o/I_o resulting in:

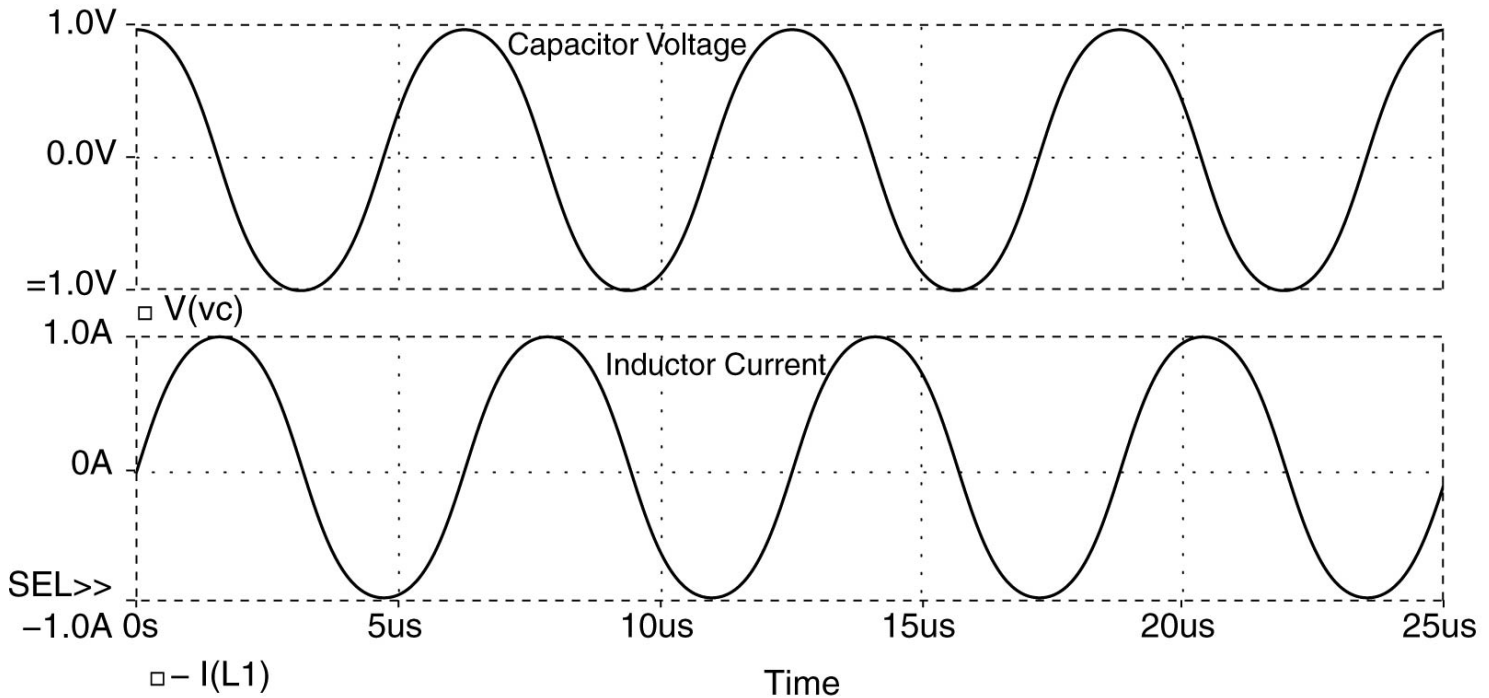
$$\frac{V_o}{I_o} = \sqrt{\frac{L}{C}} \equiv Z_o$$

The term “ Z_o ” is defined as the characteristic impedance of a resonant circuit. E.g with $C = 1$ microFarad and $L = 1$ microHenry, the resonant frequency is 10^6 radians/second (or 166.7 kHz) and that the characteristic impedance is 1 Ohm.

Simulation

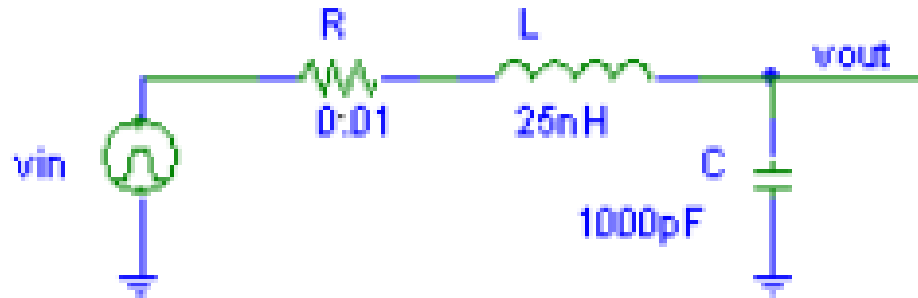


Initial conditions at $t = 0$:
capacitor voltage = 1;
inductor current = 0.



Typical Resonant Circuit

- Model of a MOSFET gate drive circuit



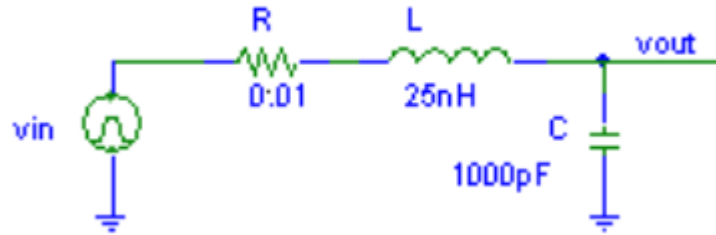
$$H(s) = \frac{v_o(s)}{v_i(s)} = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\zeta = \frac{\omega_n RC}{2} = \frac{1}{2} \frac{R}{\sqrt{\frac{L}{C}}} = \frac{1}{2} \frac{R}{Z_o}$$

Resonant Circuit --- Underdamped

- With “small” resistor, circuit is underdamped



$$\omega_n = \frac{1}{\sqrt{LC}} = 200 \text{ Mrad / sec}$$

$$f_n = 31.8 \text{ MHz}$$

$$Z_o = \sqrt{\frac{L}{C}} = 5\Omega$$

$$\zeta = \frac{\omega_n RC}{2} = \frac{1}{2} \frac{R}{\sqrt{\frac{L}{C}}} = \frac{1}{2} \frac{R}{Z_o} = 0.001$$

Resonant Circuit --- Underdamped Results

- This circuit is very underdamped, so we expect step response to oscillate at around 31.8 MHz
- Expect peaky frequency response with peak near 31.8 MHz

$$\omega_n = \frac{1}{\sqrt{LC}} = 200 \text{ Mrad / sec}$$

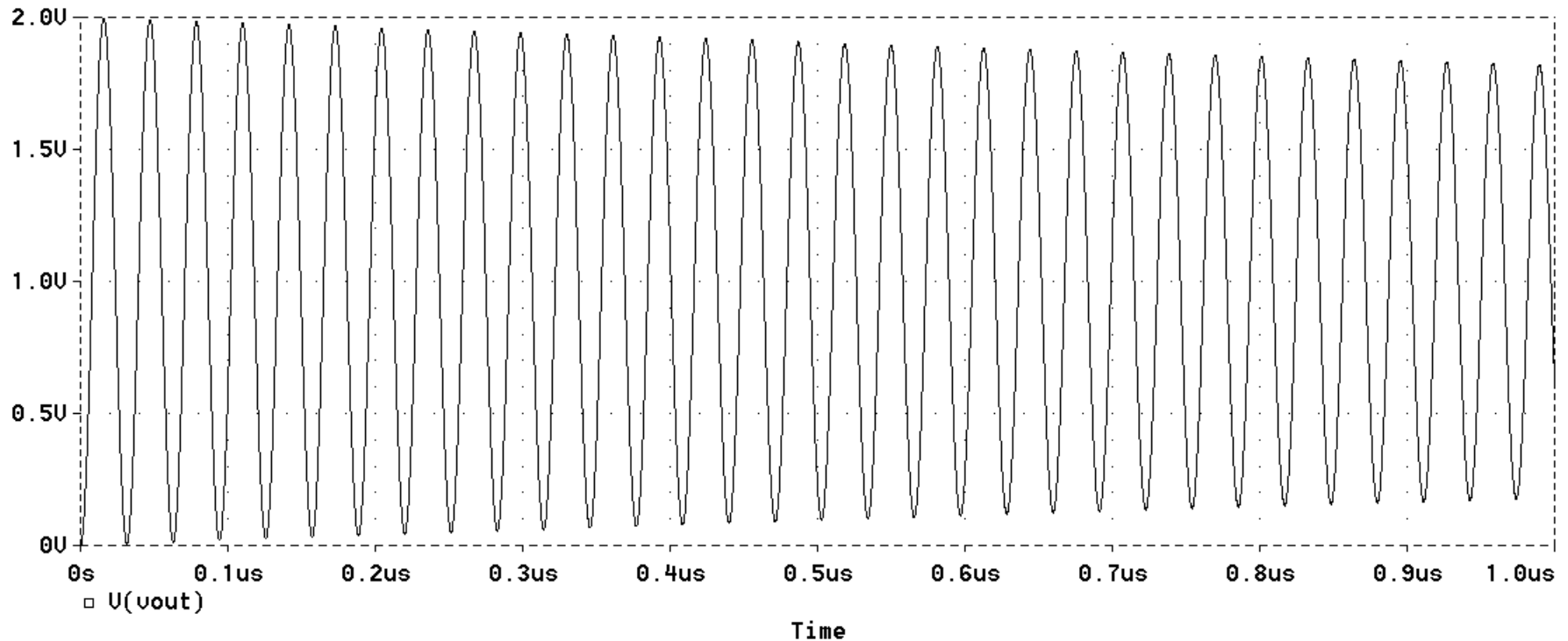
$$f_n = 31.8 \text{ MHz}$$

$$Z_o = \sqrt{\frac{L}{C}} = 5\Omega$$

$$\zeta = \frac{\omega_n RC}{2} = \frac{1}{2} \frac{R}{\sqrt{\frac{L}{C}}} = \frac{1}{2} \frac{R}{Z_o} = 0.001$$

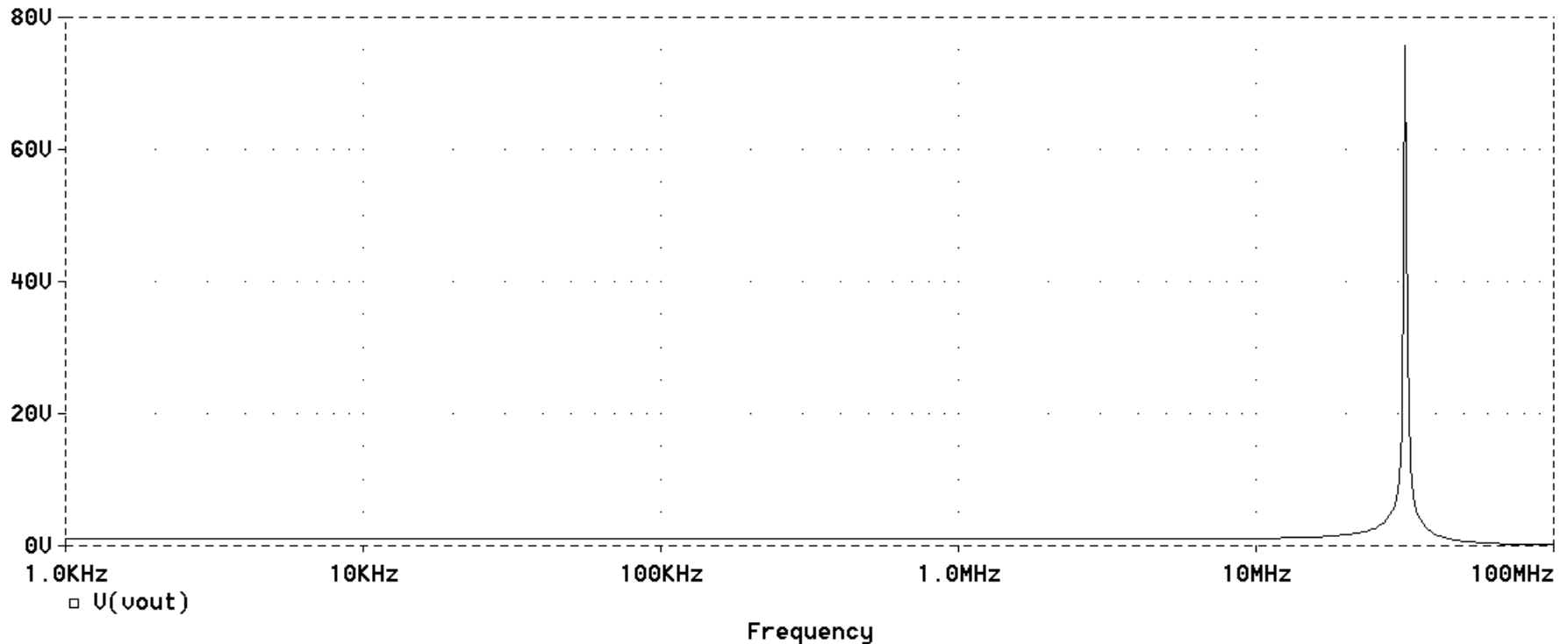
Underdamped Resonant Circuit, Step Response

- PSPICE result: rings at around 31.8 MHz



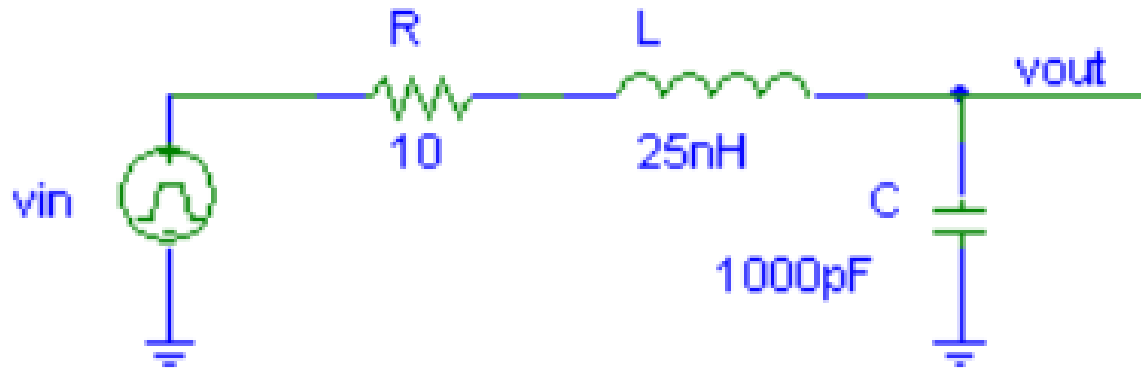
Underdamped Resonant Circuit, Frequency Response

- Frequency response peaks at 31.8 MHz



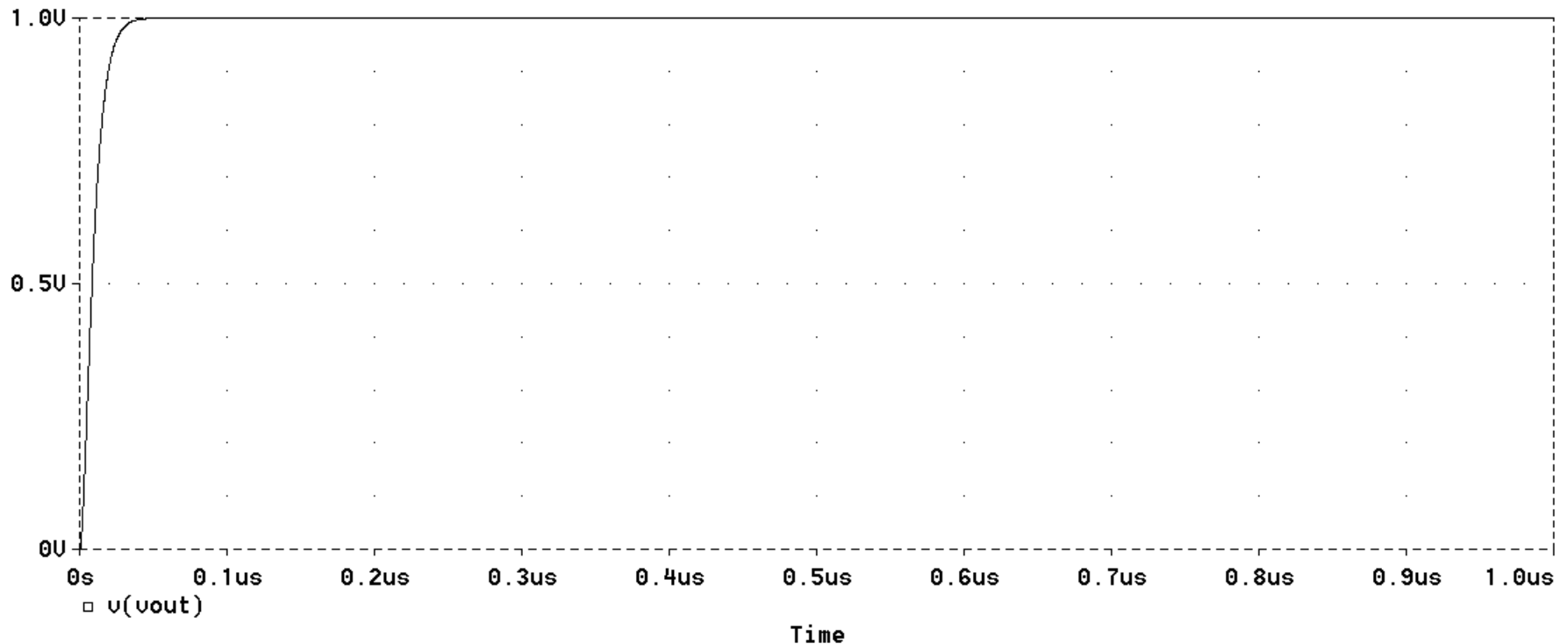
Resonant Circuit --- Critical Damping

- Now, let's employ "critical damping" by increasing value of resistor to $R = 2Z_0 = 10$ Ohms
- This is also a typical MOSFET gate drive damping resistor value



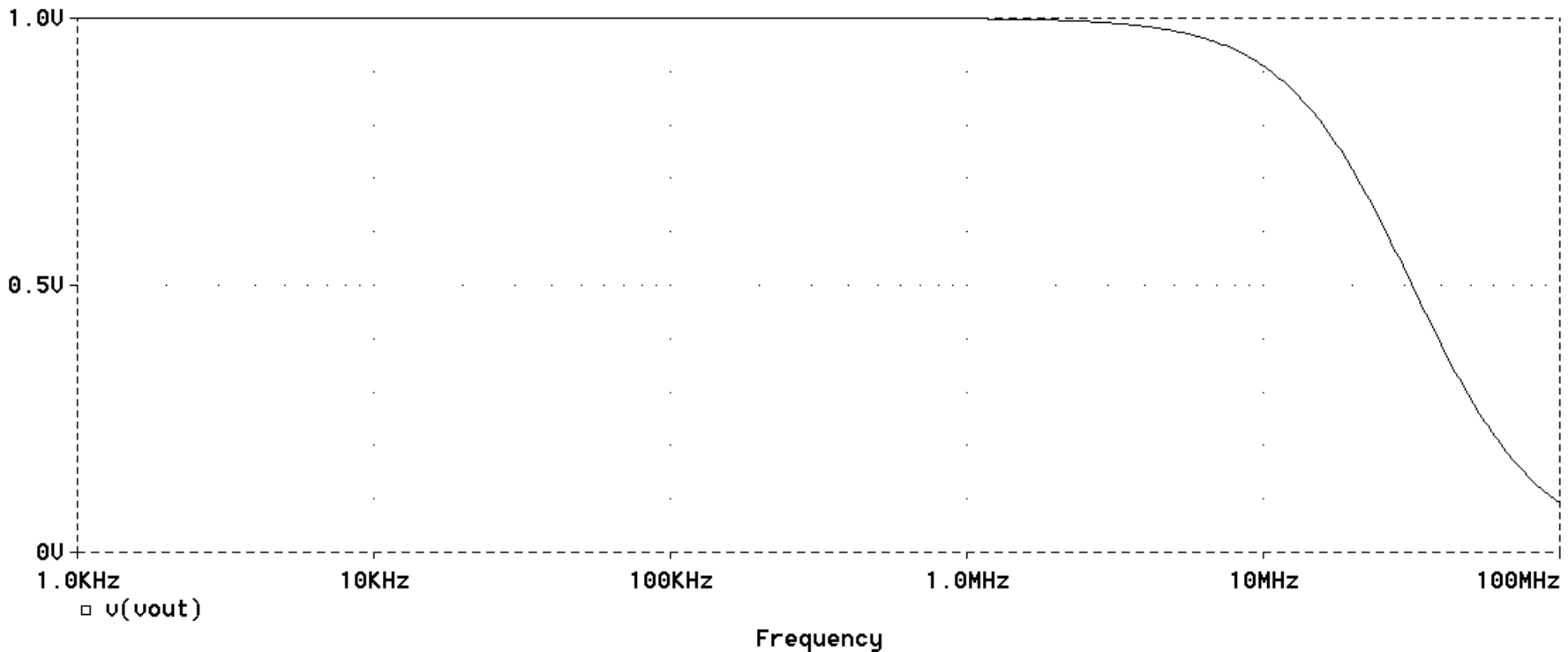
Critical Damping, Step Response

- Note that response is still relatively fast (< 100 ns response time) but with no overshoot
- “Critical damping” results in fastest step response without overshoot



Critical Damping, Frequency Response

- No overshoot in the transient response corresponds to no peaking in the frequency response



Impedance of a Series-Resonant Circuit

- The impedance is capacitive below the resonant frequency
- Impedance is inductive above resonance

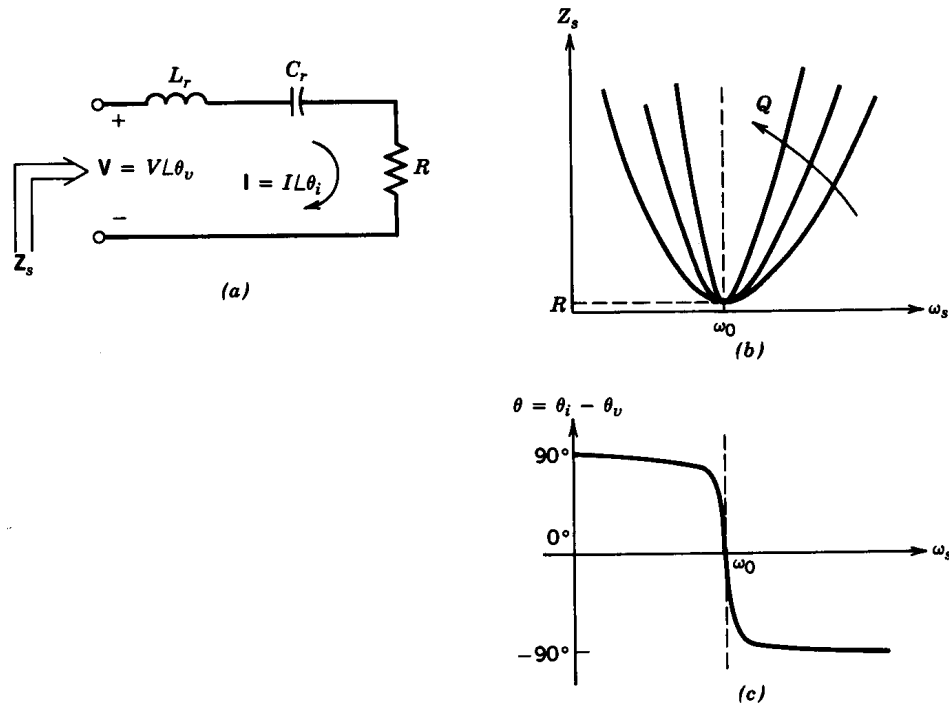


Figure 9-7 Frequency characteristics of a series-resonant circuit.

Reference: N. Mohan, *Power Electronics*, 3d edition

Impedance of a Parallel-Resonant Circuit

- The impedance is inductive below the resonant frequency

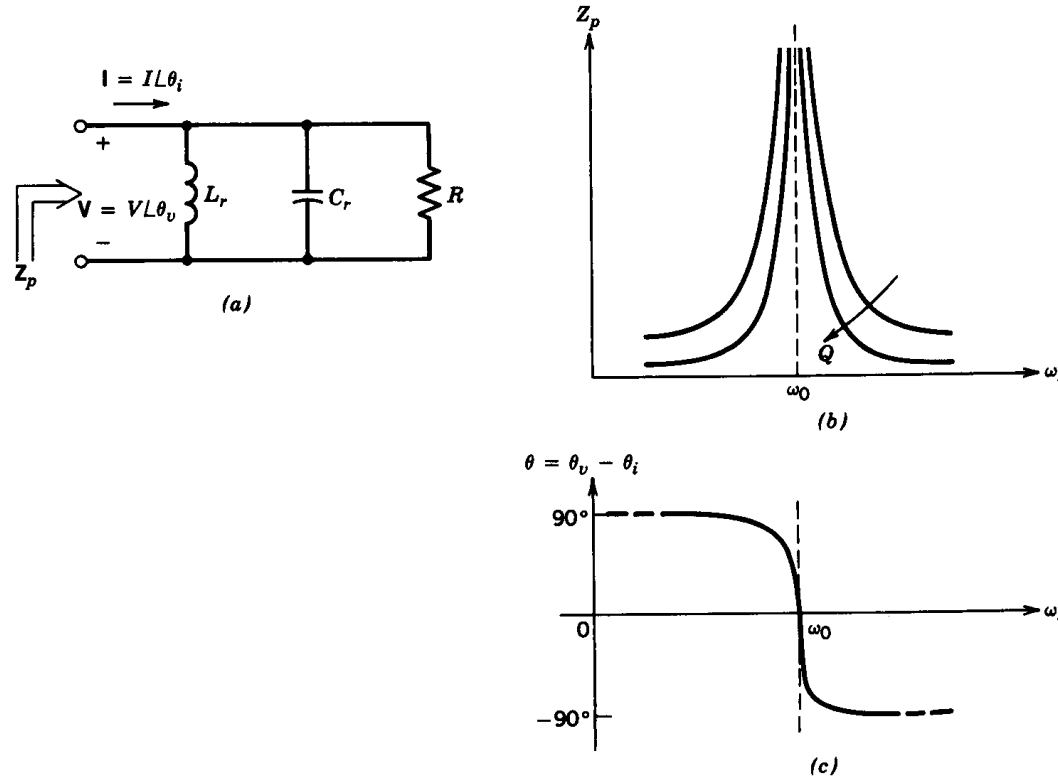


Figure 9-9 Frequency characteristics of a parallel-resonant circuit.

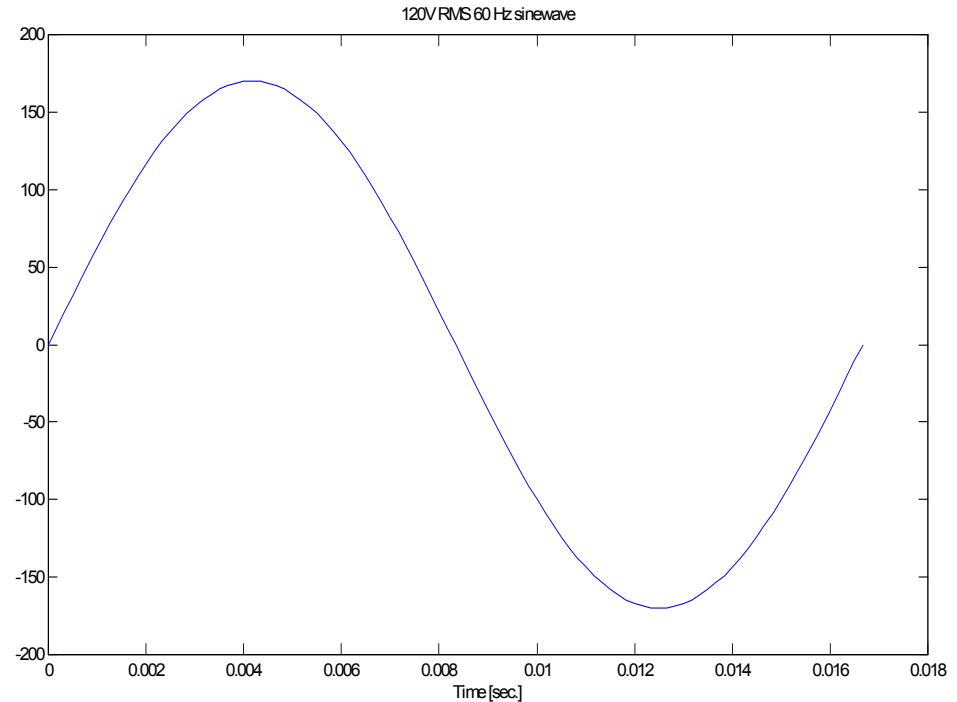
Reference: N. Mohan, *Power Electronics*, 3d edition

Power, Complex Numbers and RMS

- Basic single-phase circuits
- AC voltage, current and power
- Complex numbers
- 3-phase power
- Root-mean square (RMS)

Sinewaves

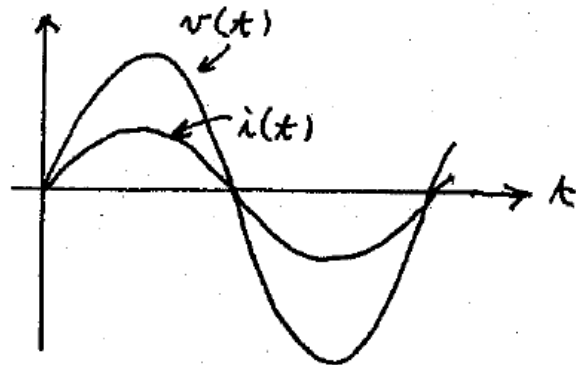
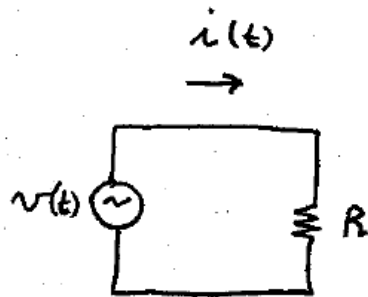
- A pure sinewave can be expressed as $v(t) = V_{pk}\sin(\omega t)$
- V_{pk} = peak voltage
- ω = radian frequency
- In Hz, $\omega = 2\pi f$ where f is in Hz
- $V_{RMS} = V_{pk}/\sqrt{2} = 120V$ for sinewave with peaks at $\pm 170V$
- More on RMS later



Sinewave Voltage Source with Resistive Load

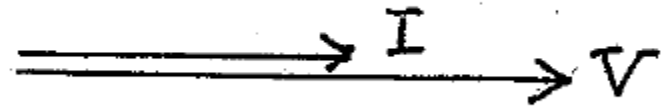
- $v(t)$ and $i(t)$ are in phase and have the same shape; i.e. no harmonics in current

- Time representation



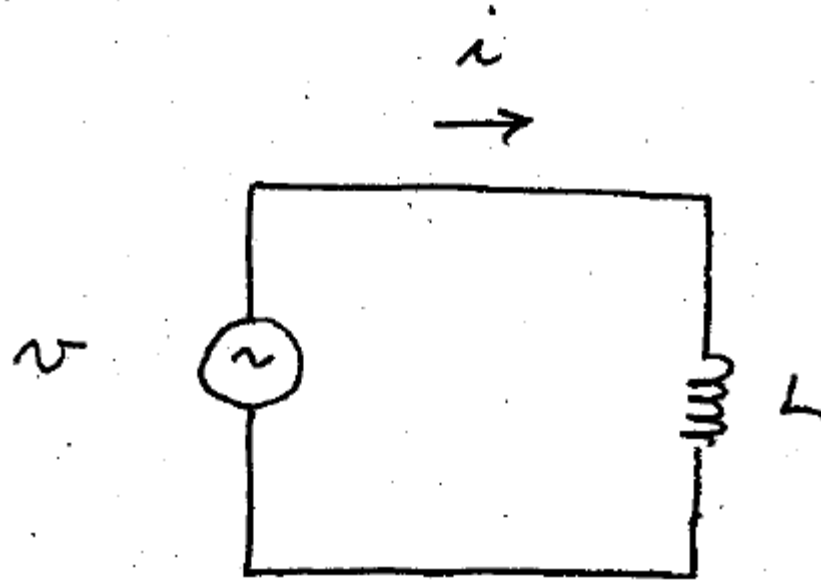
- Phasor representation

-In this case, V and I have the same phase



Sinewave with Inductive Load

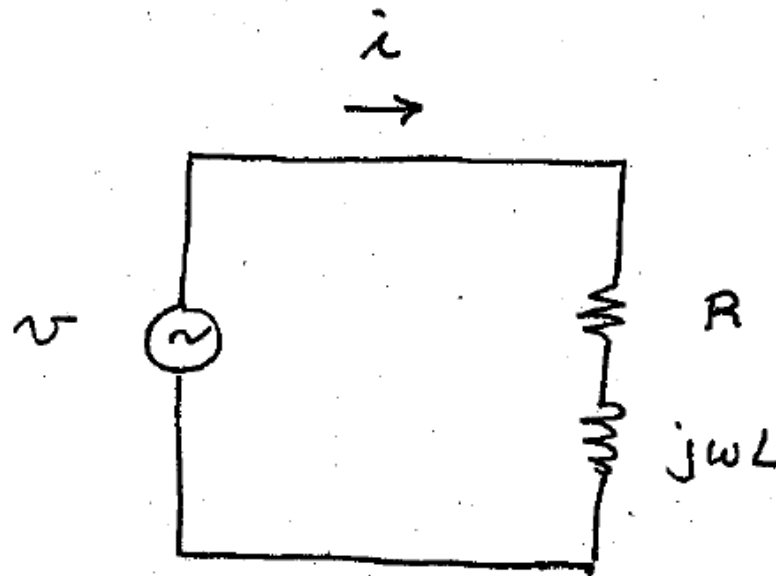
- For an inductor, remember that $v = L di/dt$
- So, $i(t)$ lags $v(t)$ by 90° in an inductor



Sinewave with L/R Load

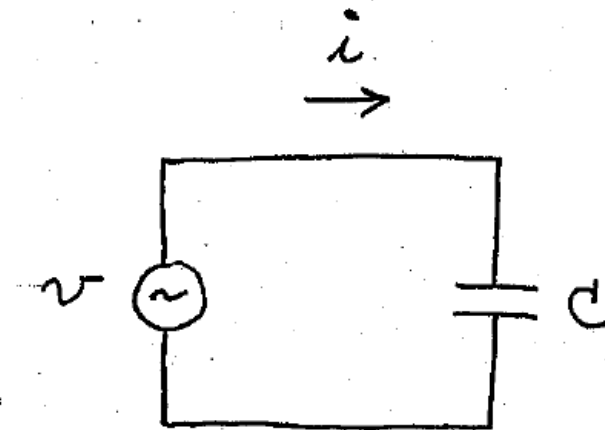
- Phase shift (also called angle) between v and i is somewhere between 0° and -90°

$$\angle = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

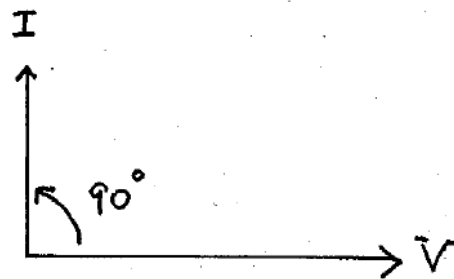


Sinewave with Capacitive Load

- Remember that $i = Cdv/dt$ for a capacitor
- Current leads voltage by $+90^\circ$



- Phasor representation



Phasor Representation of L and C

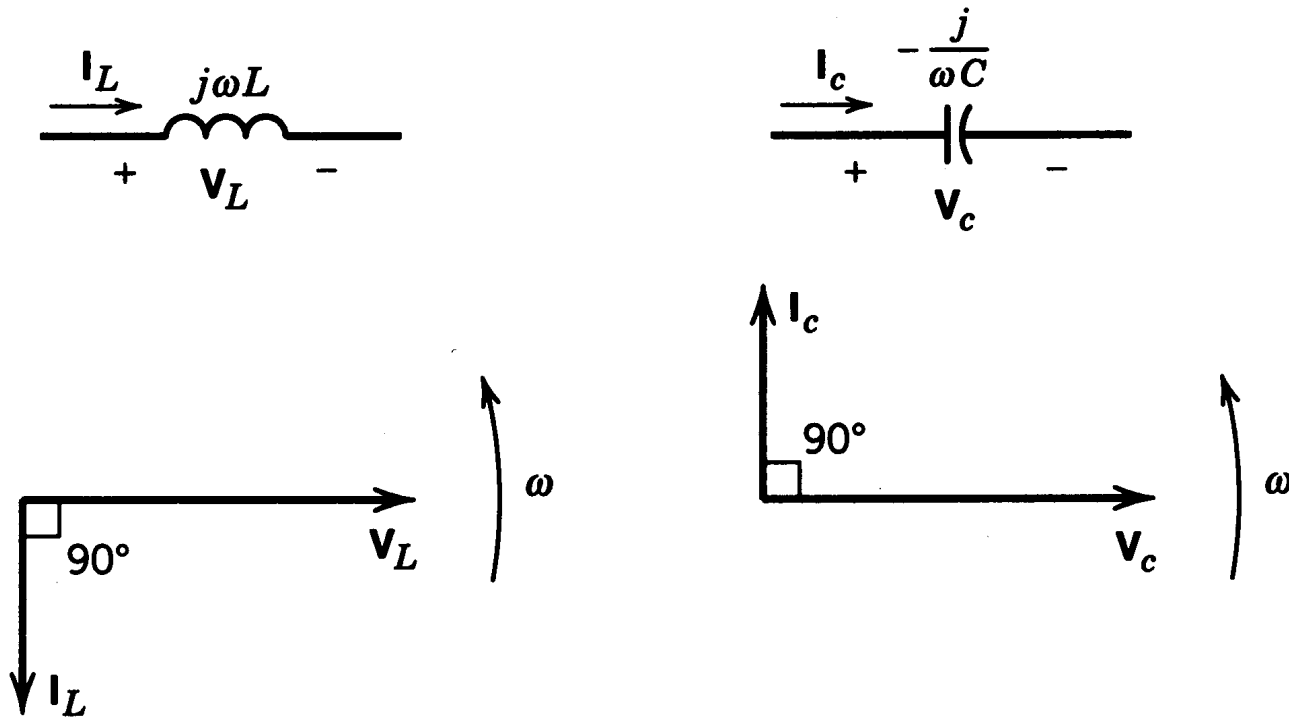


Figure 3-6 Phasor representation.

In inductor, current lags voltage by 90 degrees

In capacitor, voltage lags current by 90 degrees

Reference: N. Mohan, *Power Electronics*, 3d edition

Response of L and C to pulses

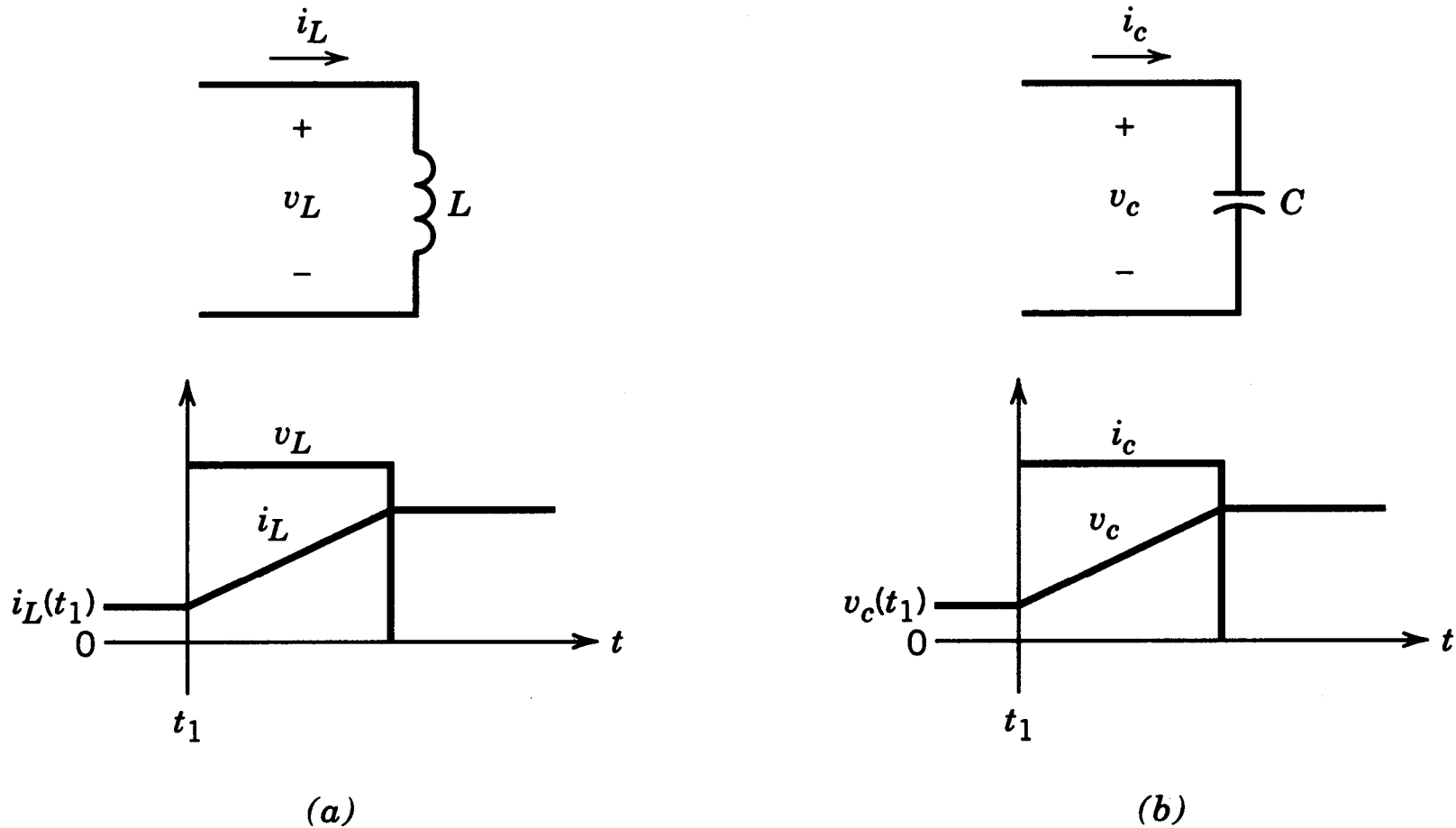
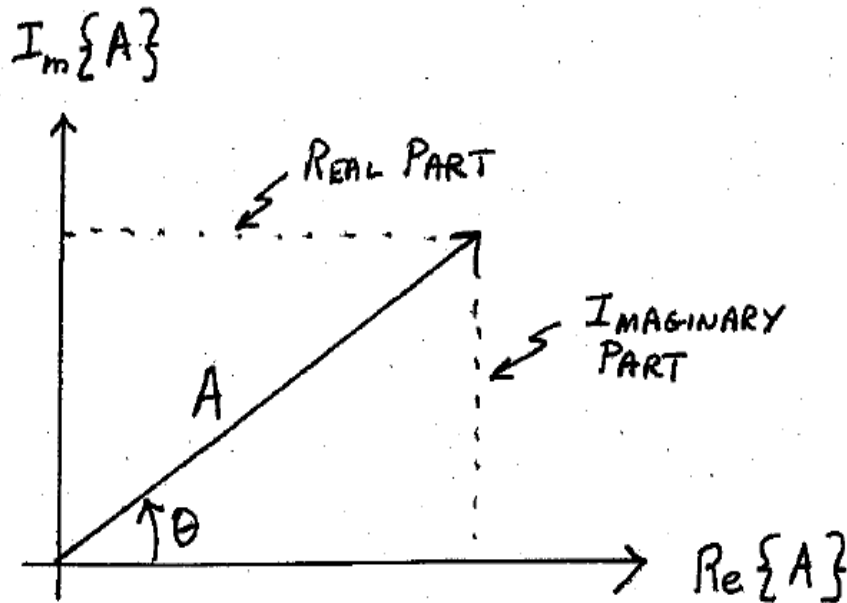


Figure 3-7 Inductor and capacitor response.

Reference: N. Mohan, *Power Electronics*, 3d edition

Review of Complex Numbers

- In “rectangular” form, a complex number is written in terms of real and imaginary components
- $A = \text{Re}(A) + j \times \text{Im}(A)$



- Angle

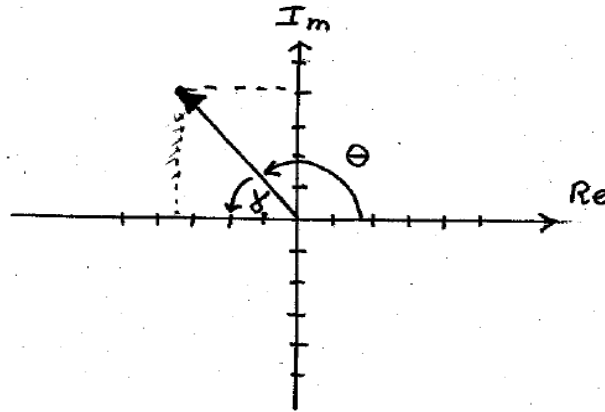
$$\theta = \tan^{-1} \left(\frac{\text{Im}(A)}{\text{Re}(A)} \right)$$

- Magnitude of A

$$|A| = \sqrt{(\text{Re}(A))^2 + (\text{Im}(A))^2}$$

Find Polar Form

- Assume that current $I = -3.5 + j(4.2)$



$$|I| = \sqrt{(-3.5)^2 + (4.2)^2} = 5.5A$$

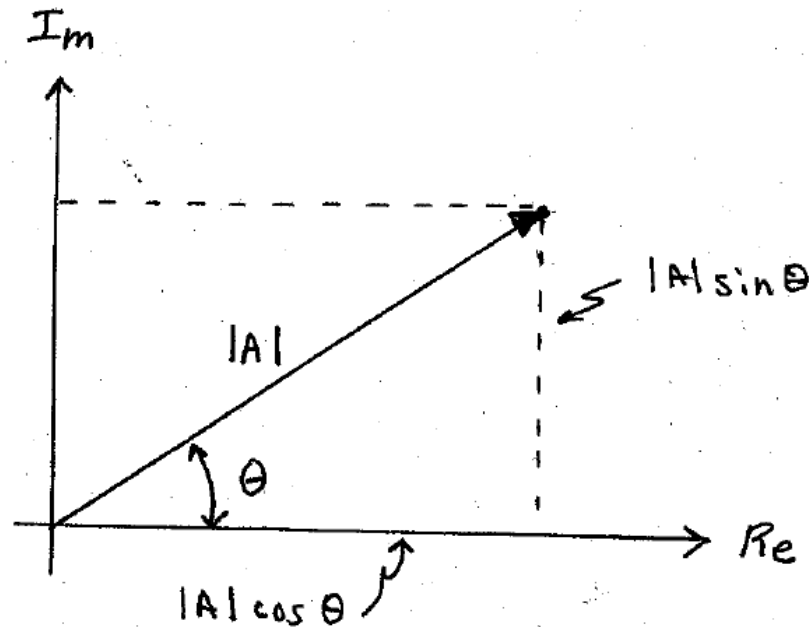
$$\theta = 180^\circ - \gamma$$

$$\gamma = \tan^{-1}\left(\frac{4.2}{3.5}\right) = 50.2^\circ$$

$$\theta = 180^\circ - 50.2^\circ = 129.8^\circ$$

$$\therefore I = 5.5A \angle 129.8^\circ$$

Converting from Polar to Rectangular Form



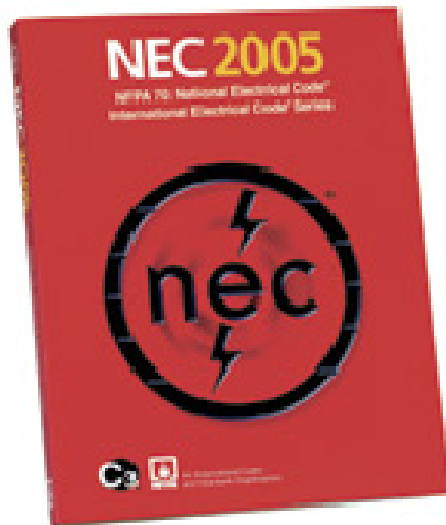
$$\text{Re}\{A\} = |A| \cos(\theta)$$

$$\text{Im}\{A\} = |A| \sin(\theta)$$

National Electrical Code (NEC)

- Put out by National Fire Protection Agency (NFPA) as NFPA 70
- Initially developed in 1897, and is updated every 3 years
- Sets out requirements for wire sizing (phase conductors, neutrals, grounds) and other rules for installation of power in residential single phase and industrial 3 phase systems

National Electrical Code® Softbound 2005 Edition (NFPA 70)



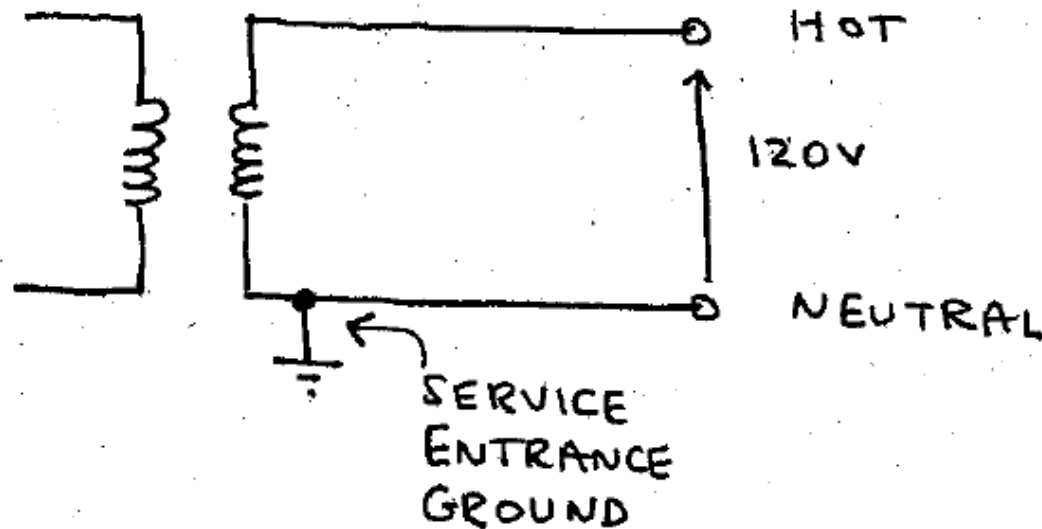
Item # 7005SB

Format	List	Member	Cart
Book	\$72.50	\$65.25	ADD
PDF	\$72.50	\$65.25	ADD

[View Cart](#) [Check Out](#)

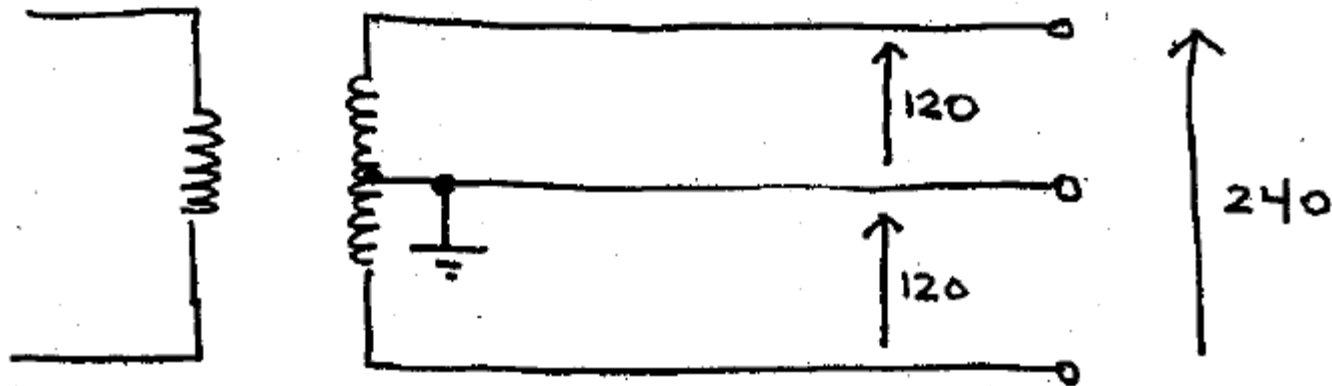
120V, 2-Wire System

- Older system, largely (hopefully) replaced in residences
- Neutral = “grounded conductor”
- Voltage between hot and neutral is 120 VAC (or 120 VRMS)
- Voltage stepped down at pole-mount or pad-mount transformer; enters house service entrance at 120V between HOT and NEUTRAL



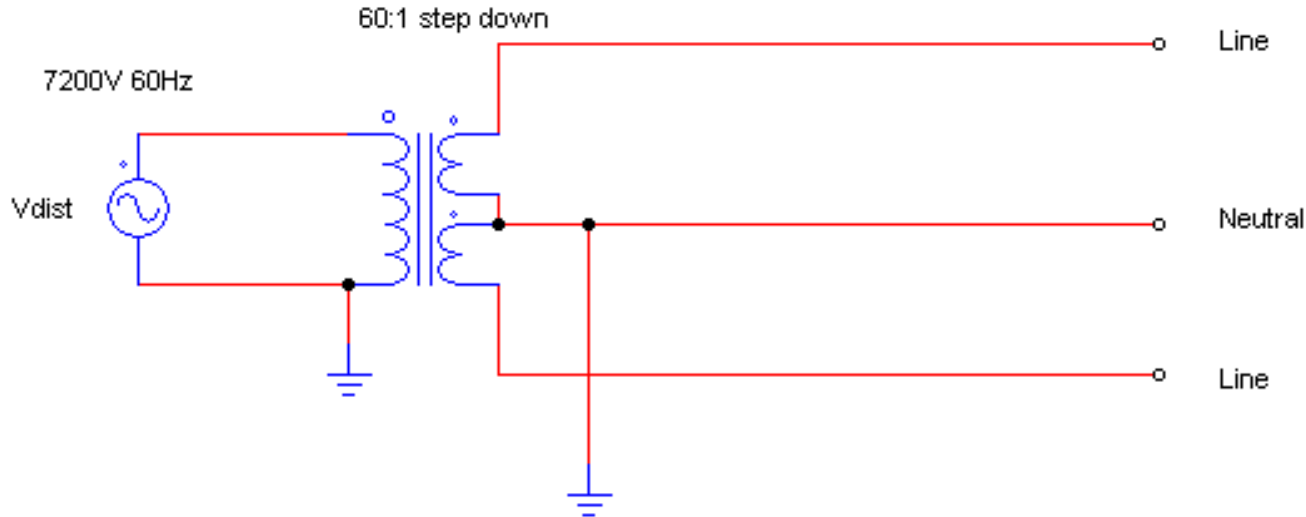
120/240, 3-Wire System

- Most common residential service; 3 wires are derived by center-tapping the distribution transformer secondary
- Voltage between one phase and neutral is 120V
- Voltage between phases is 240V: used for hot water heaters, dryers, air conditioners, etc.
- Neutral is grounded at the service entrance



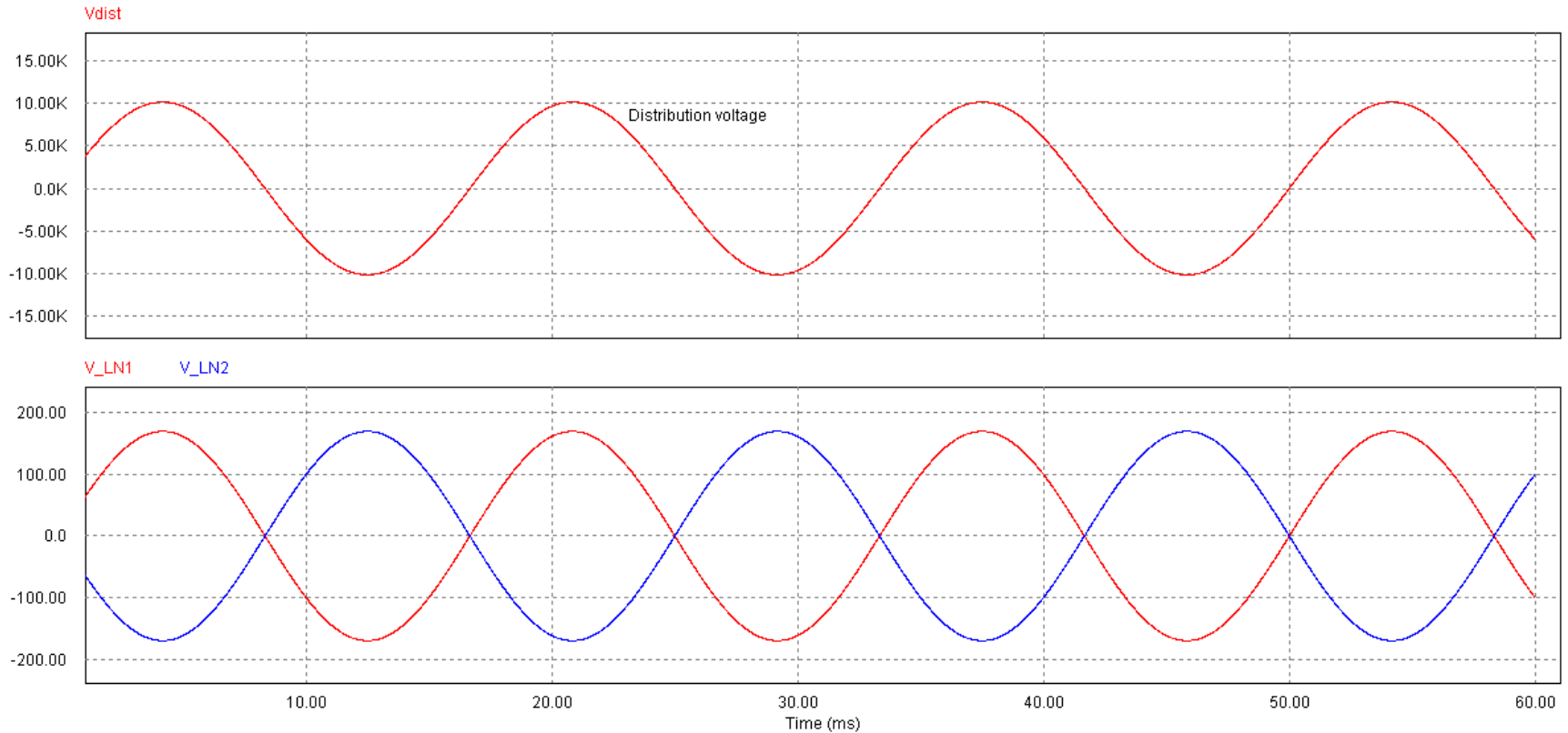
Typical Single-Phase Residential Service

- Typical 7.2 kV distribution voltage
- Output voltage is 120V with 2 single phases
- You'll run low-power loads (i.e. lights, electronics) from a single phase
- High power loads (i.e. clothes dryer, hot water heater) run phase-to-phase



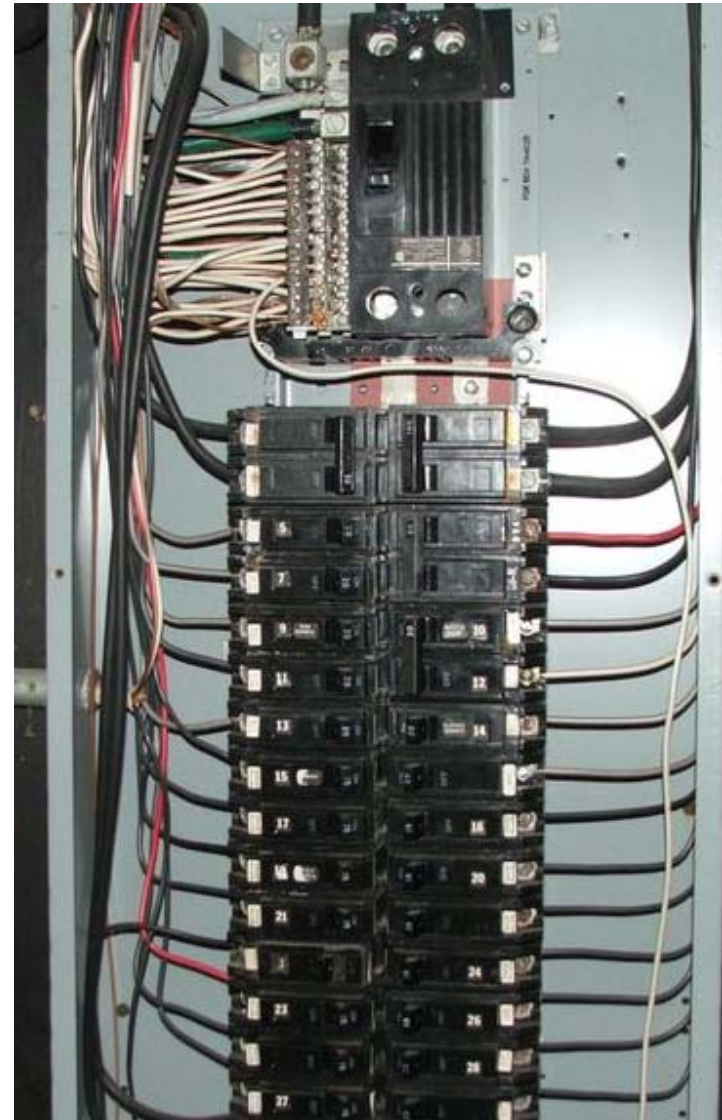
Single-Phase Service --- Simulation Results

- Transformer step-down



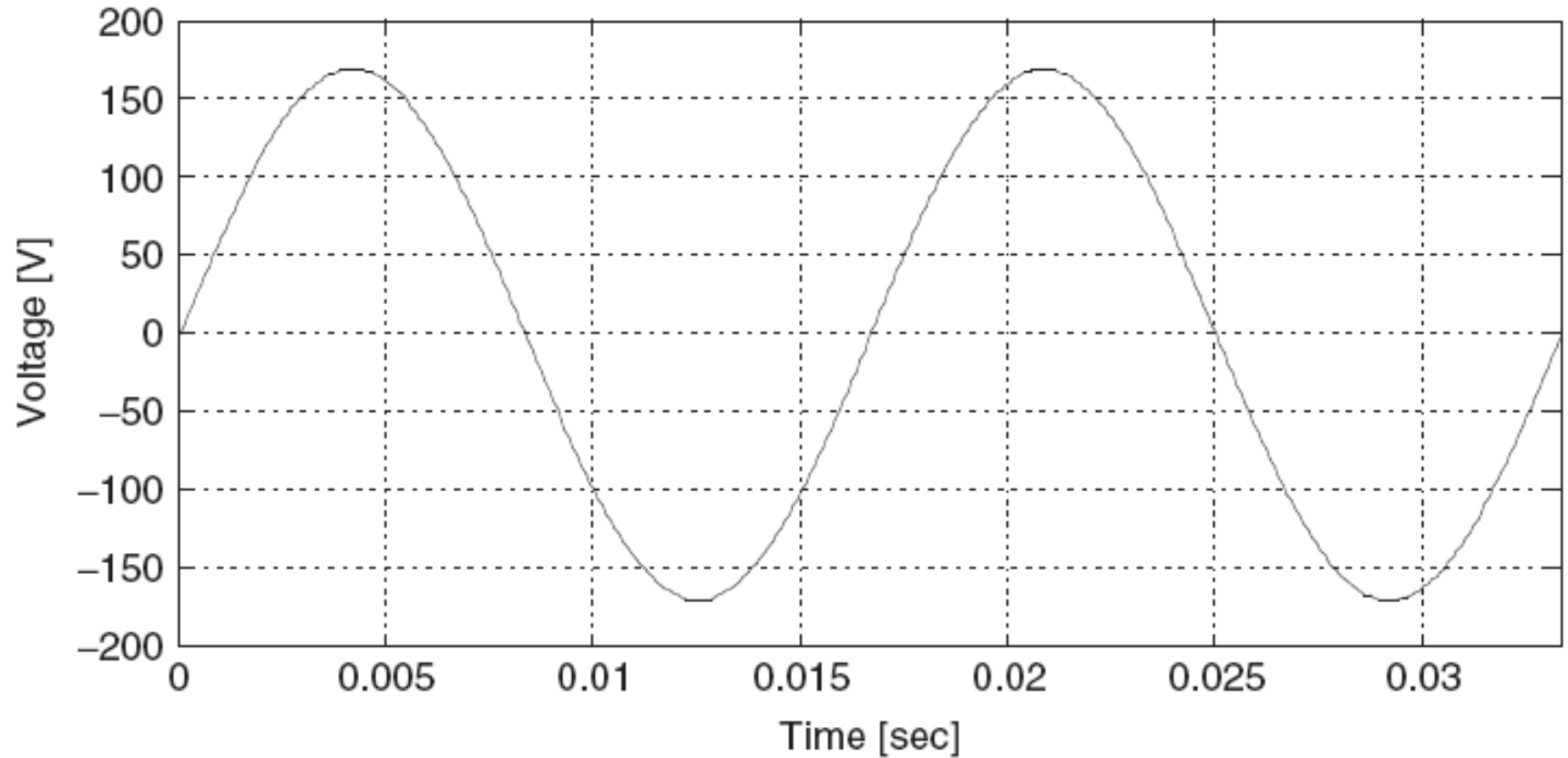
200A Breaker Panel

- Main breaker at top (200A)
- Multiple branch circuits below: black = hot wire, white = neutral



Single-Phase 60 Hz Power

Ideal 60 Hz sinewave



Reference: A. Kusko and M. Thompson, *Power Quality in Electrical Systems*, McGraw-Hill, 2007

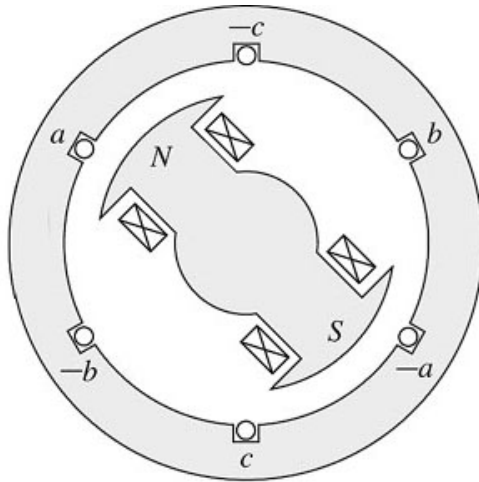
Three-Phase Power

- Three-phase is used in higher power applications
- You can carry more power with less copper using 3 phase
- Large generators and motors are more efficient and smaller using 3-phase compared to single-phase
- 3-phase voltages are created by 3-phase generators

Three-Phase Generators

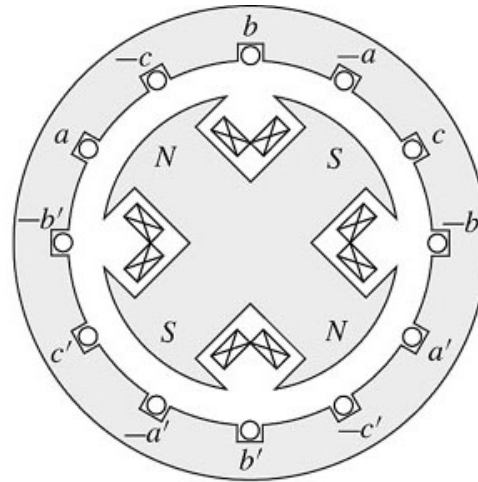
- More on 3-phase motor/generators later

2-pole



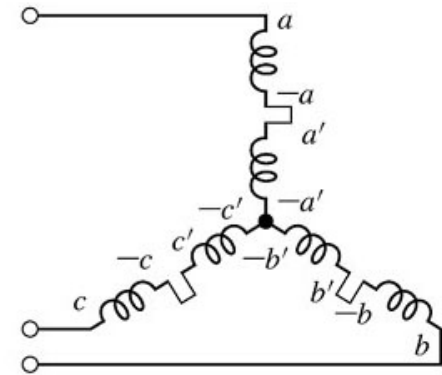
(a)

4-pole



(b)

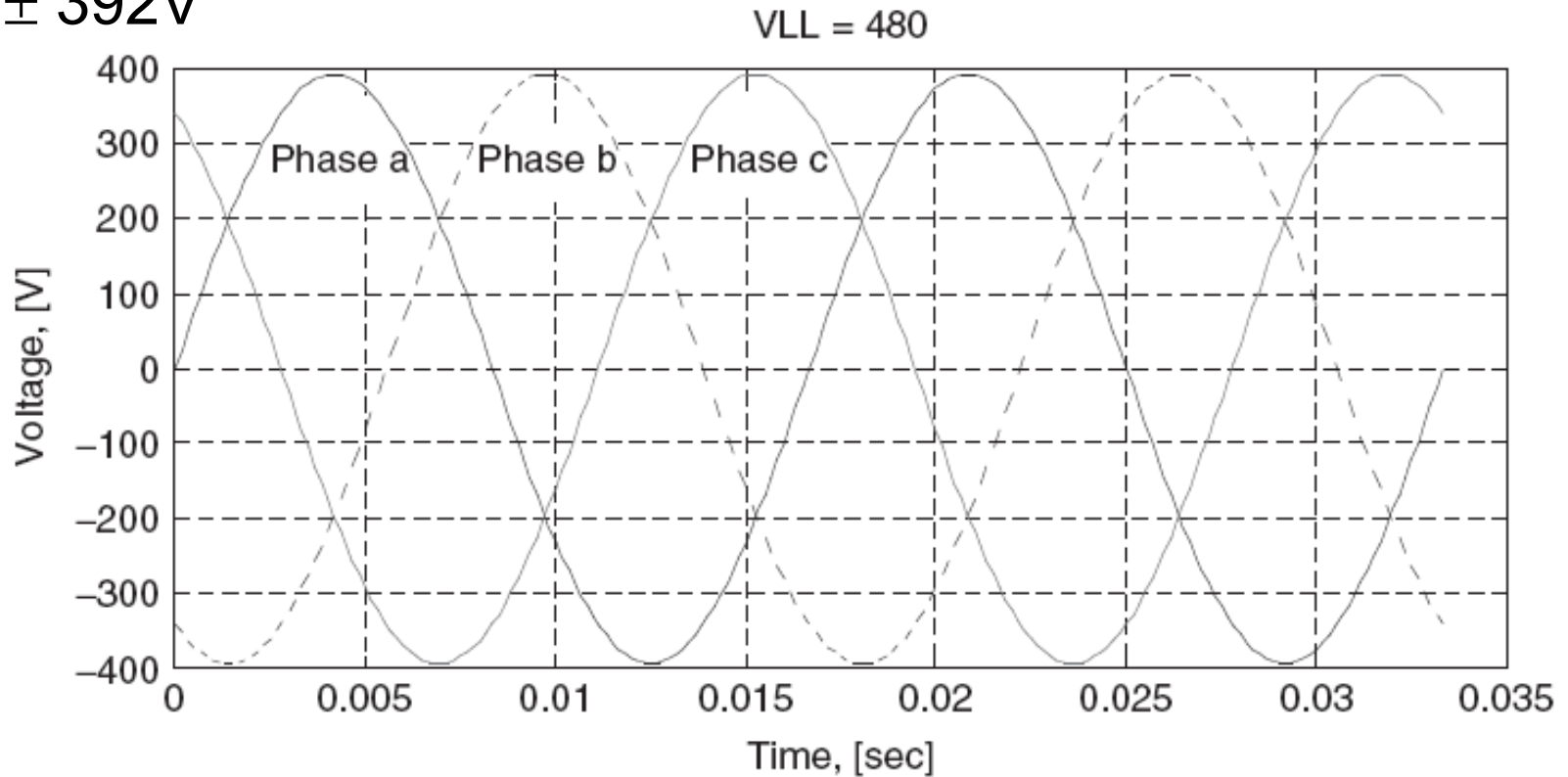
Y-connection
of windings



(c)

Three-Phase, 480V 60 Hz Power

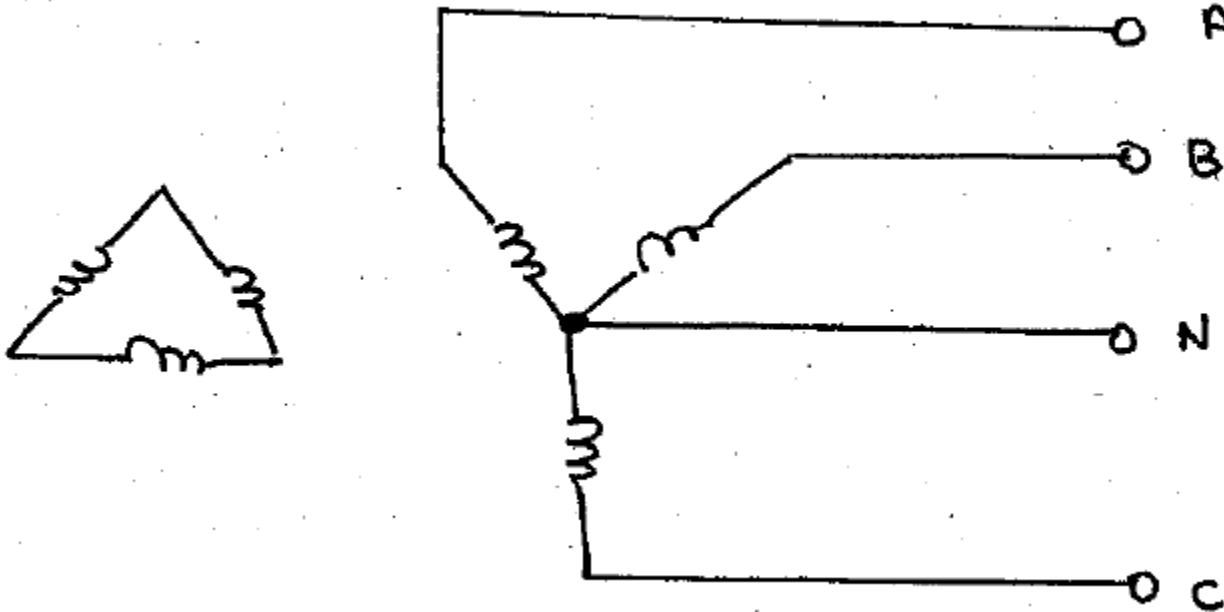
- Line-line voltage is 480V RMS
- Line-neutral voltage is 277V (or $480/\sqrt{3}$), which has peaks at $\pm 392V$



Reference: A. Kusko and M. Thompson, *Power Quality in Electrical Systems*, McGraw-Hill, 2007

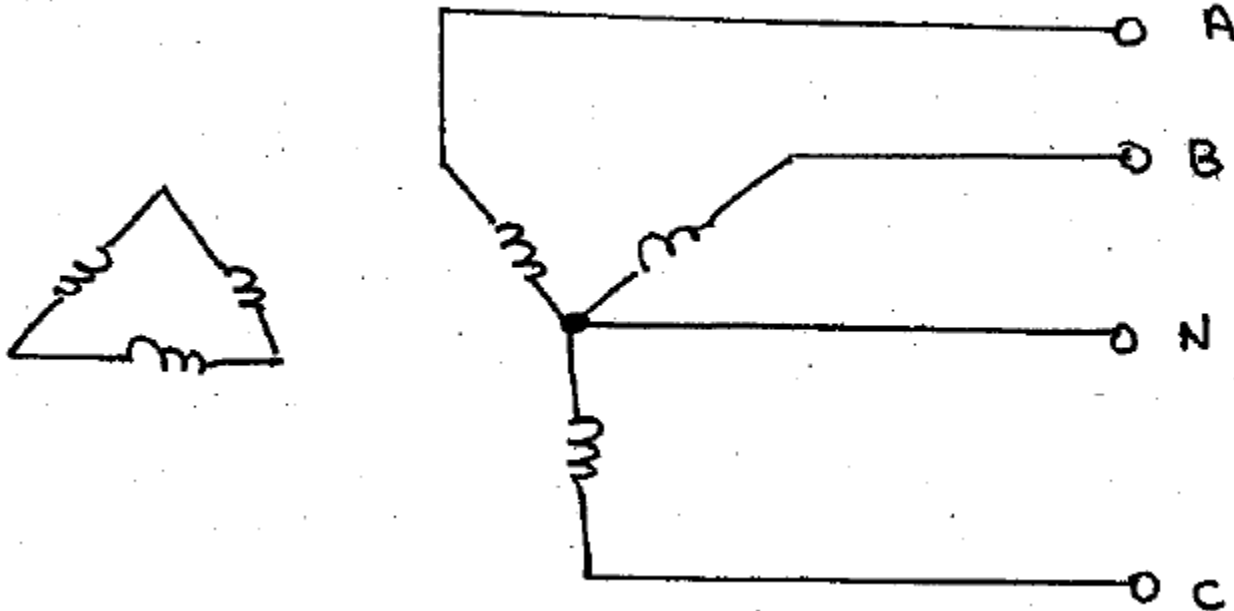
208Y/120, 3-Phase, 4 Wire

- Can drive small 3 phase loads or single phase loads
- Voltage phase-phase = 208V (i.e. from A to B)
- Voltage phase-neutral = $208/\sqrt{3} = 120V$



480/277, 3-Phase, 4 Wire

- Typical industrial voltage for larger 3-phase loads
- Large 3-phase loads can be connected to the three phases
- Smaller 277V single-phase loads can be connected between phase and ground
- Phase-phase = 480V; phase-neutral = 277V



480V 4-Wire System

- Line-line = 480V; line-neutral = 277V

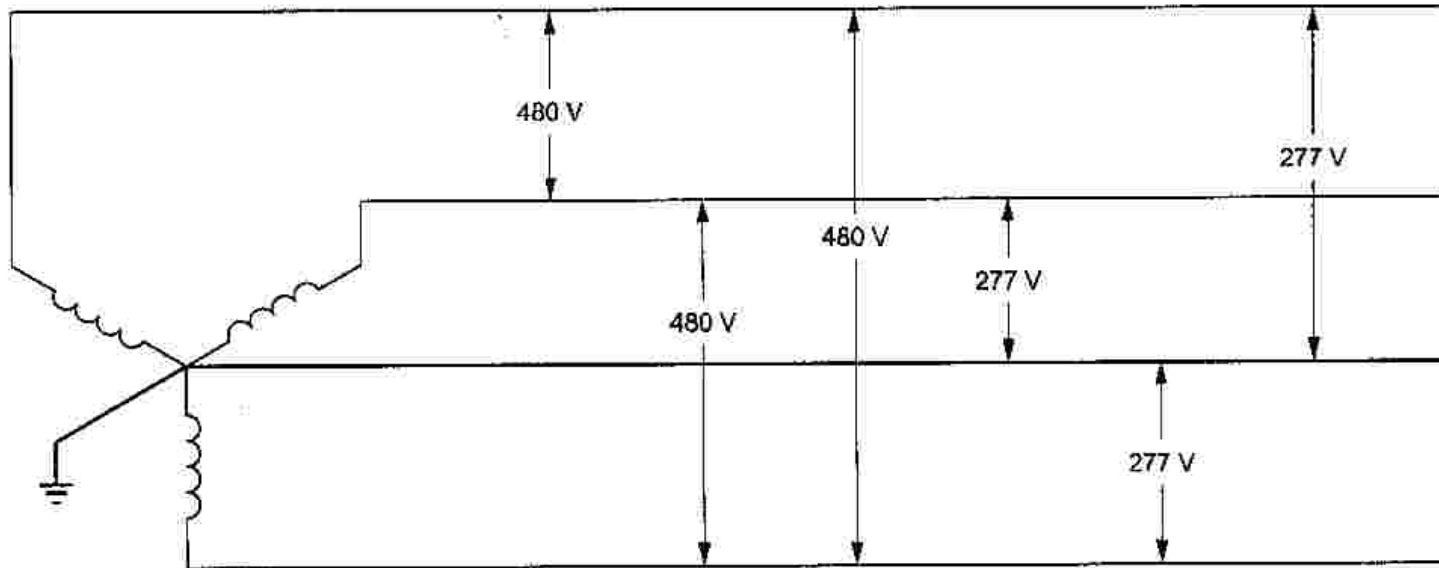


FIGURE 1-1 480Y/277-Volt System

Reference: Ralph Fehr, *Industrial Power Distribution*, Prentice Hall, 2002

Some Comments on Delivery Voltage

- A typical factory utilization voltage is 480Y/277V, meaning that a 4-wire Y connected service is provided with 480V line-line and a line-neutral voltage of 277V
- Service for industrial facilities can be supplied by a utility at distribution voltage (2.4kV to 34.5 kV) or at subtransmission or transmission voltages (46 kV to 230 kV)
- 1888: Polyphase AC system invented by Tesla
- First three-phase system was installed between Lauffen and Frankfurt am Main in Germany in 1891

Root Mean Square (RMS)

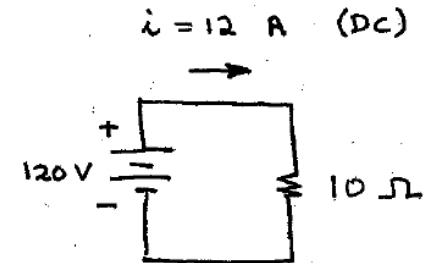
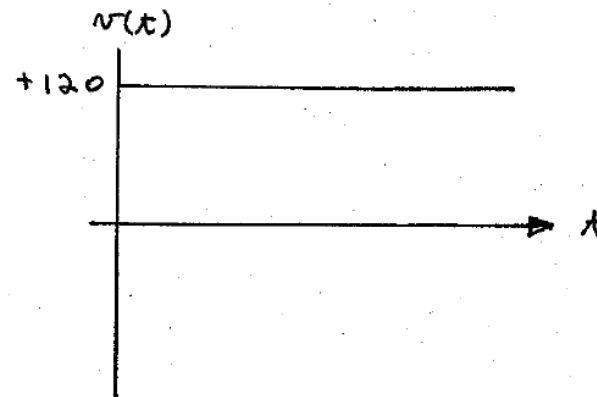
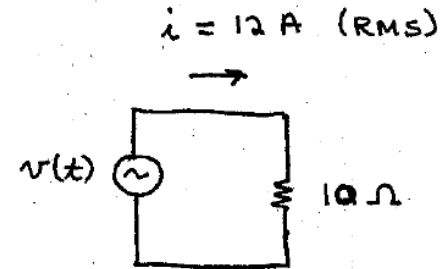
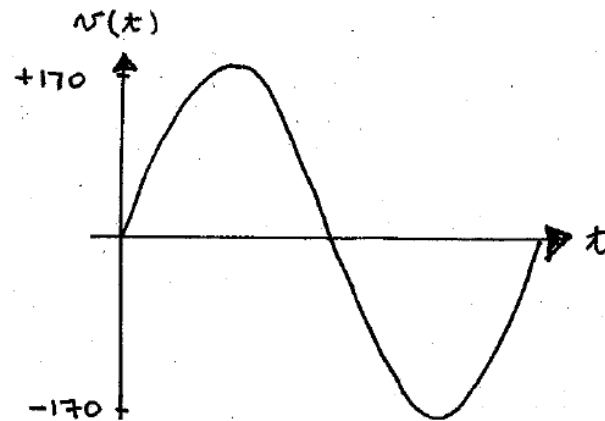
- Used for description of periodic, often multi-harmonic, waveforms
- Calculation of RMS is done by taking the square root of the average over a cycle (mean) of the square of a waveform

$$I_{RMS} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

- RMS current of any waveshape will dissipate the same amount of heat in a resistor as a DC current of the same value
 - DC waveform: $V_{rms} = V_{DC}$
 - Symmetrical square wave: $I_{RMS} = I_{pk}$
 - Pure sine wave: $I_{RMS} = 0.707 I_{pk}$
- Example: 120 VRMS line voltage has peaks of ± 169.7 V

Intuitive Description of RMS

- The RMS value of a sinusoidal or other periodic waveform dissipates the same amount of power in a resistive load as does a battery of the same RMS value
- So, 120VRMS into a resistive load dissipates as much power in the load as does a 120V battery

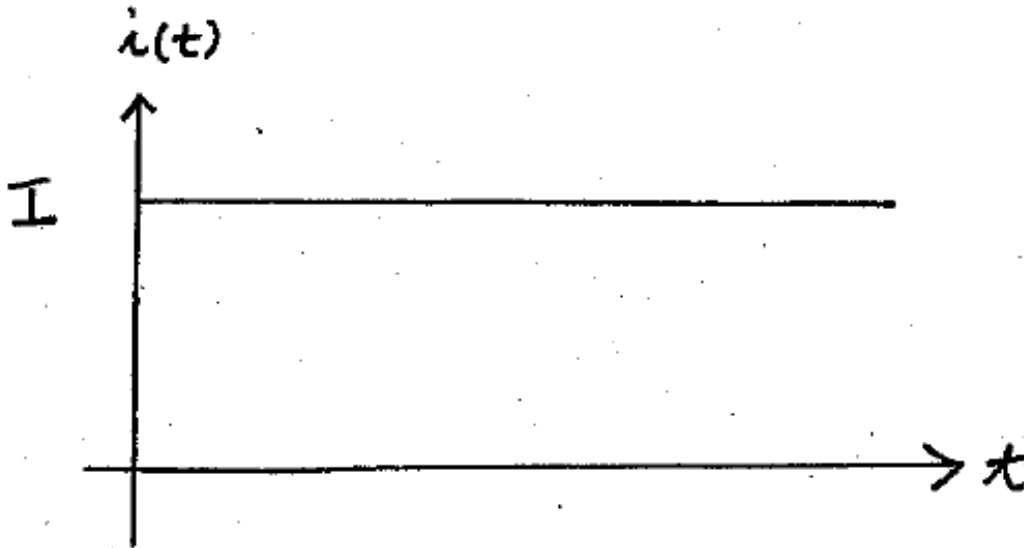


RMS Value of Various Waveforms

- Following are waveforms typically found in power electronics, power systems, and motors, and their corresponding RMS values
- Reference: R. W. Erickson and D. Maksimovic, *Fundamentals of Power Electronics*, 2nd edition, Kluwer, 2001

DC Voltage

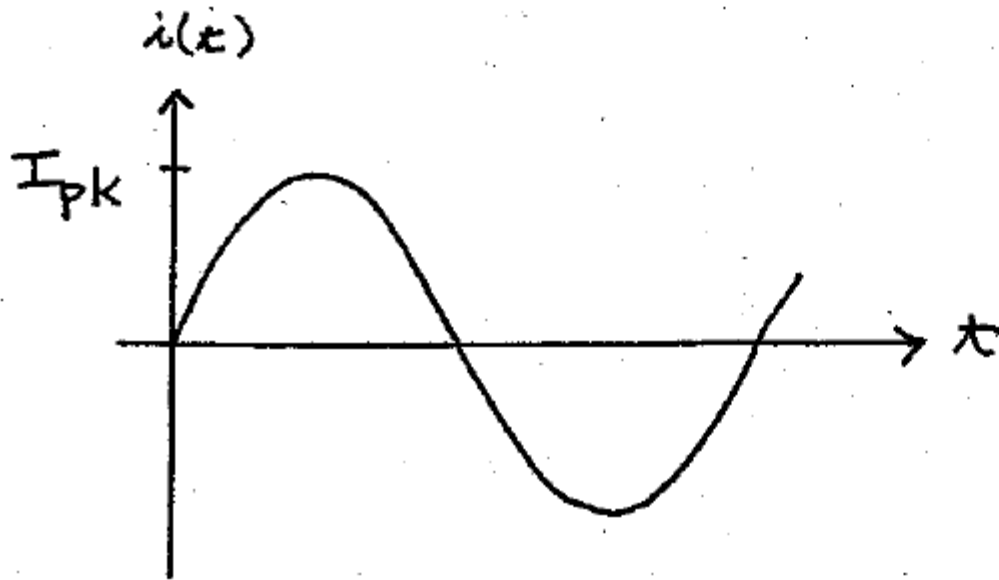
- Battery



$$\text{RMS} = I$$

Pure Sinewave

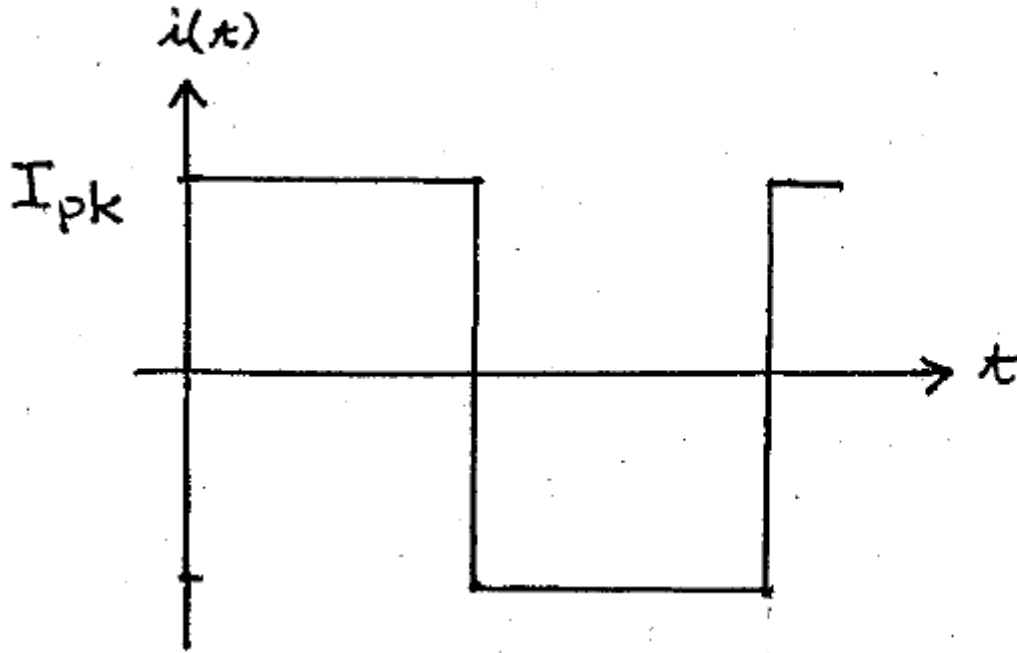
- AC line



$$RMS = \frac{I_{pk}}{\sqrt{2}}$$

Square Wave

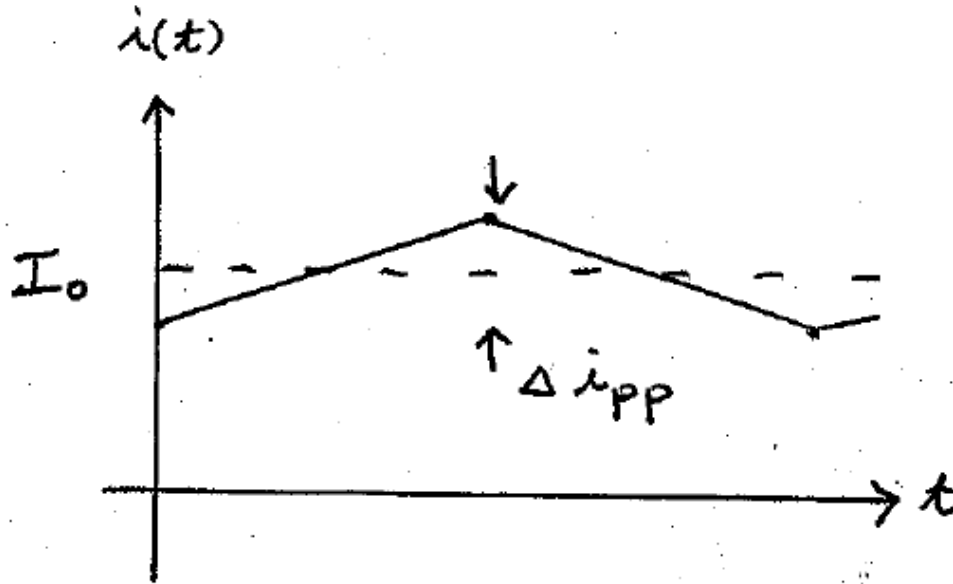
- This type of waveform can be put out by a square wave converter or full-bridge converter



$$RMS = I_{pk}$$

DC with Ripple

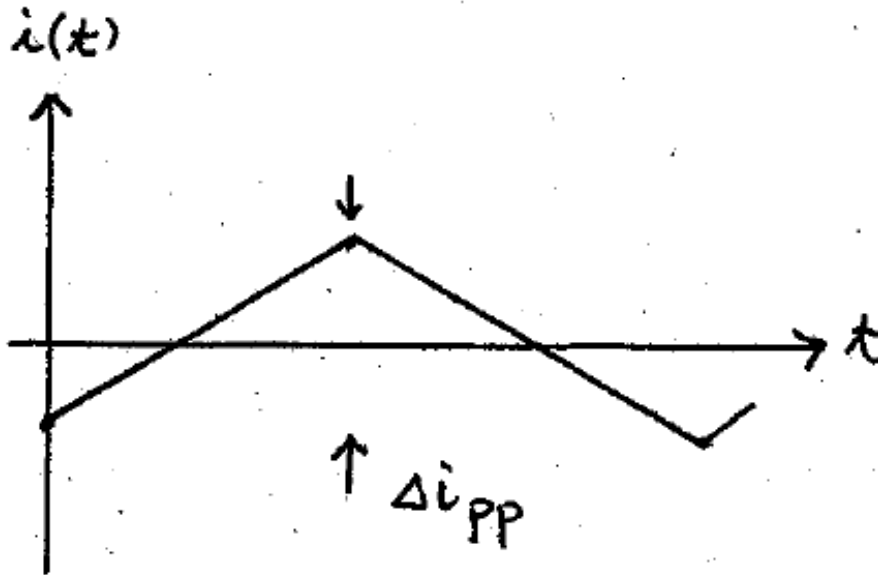
- Buck converter inductor current (DC value + ripple)



$$RMS = I_o \sqrt{1 + \left(\frac{1}{3}\right) \left(\frac{\Delta i_{pp}}{2I_o}\right)^2}$$

Triangular Ripple, No DC Value

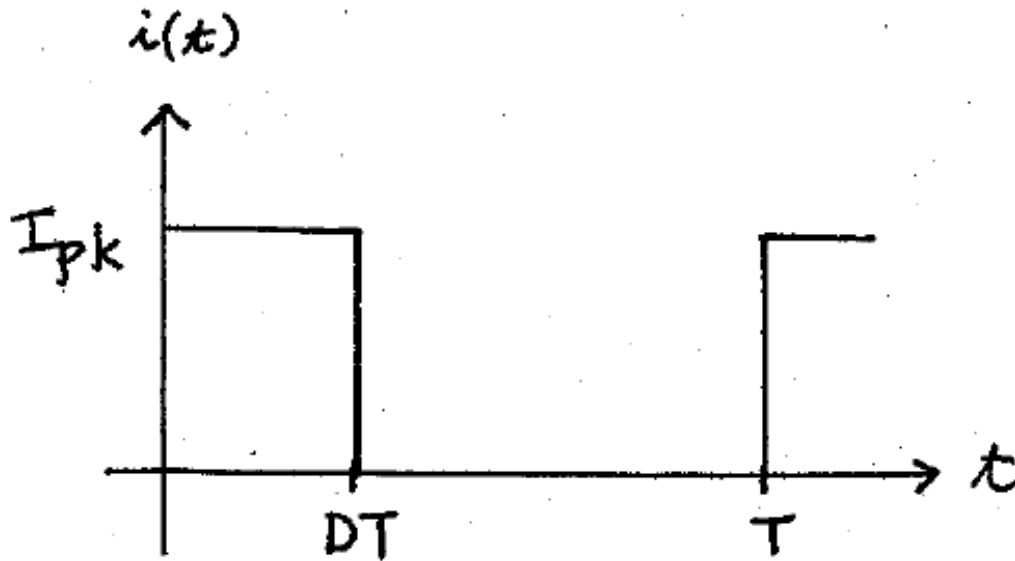
- Capacitor ripple current in some converters (no DC value)



$$RMS = \frac{\Delta i_{pp}}{2\sqrt{3}}$$

Pulsating Waveform

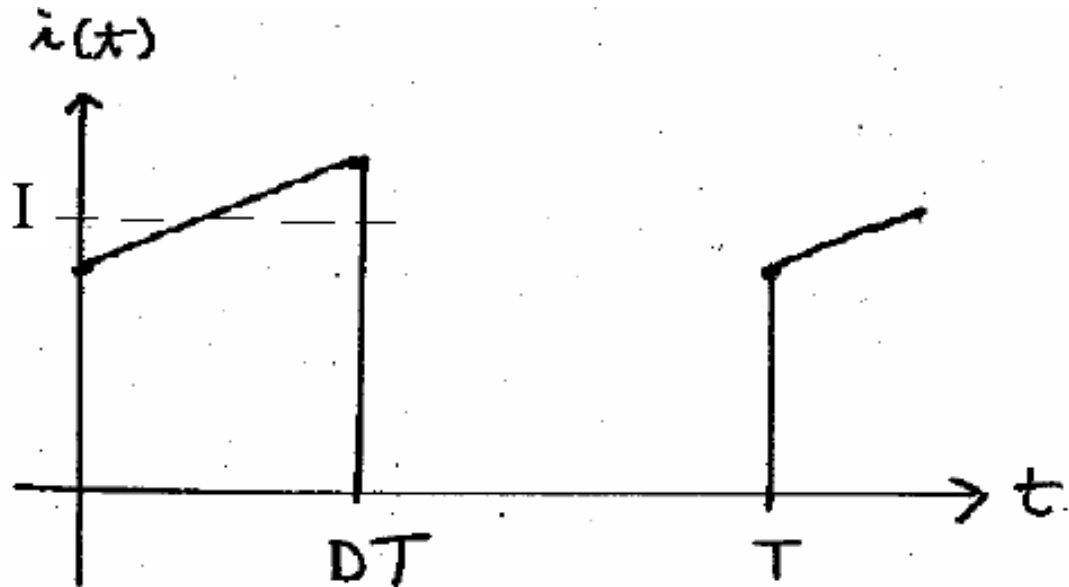
- Buck converter input switch current (assuming small ripple)



$$RMS = I_{pk} \sqrt{D}$$
$$0 < D < 1$$

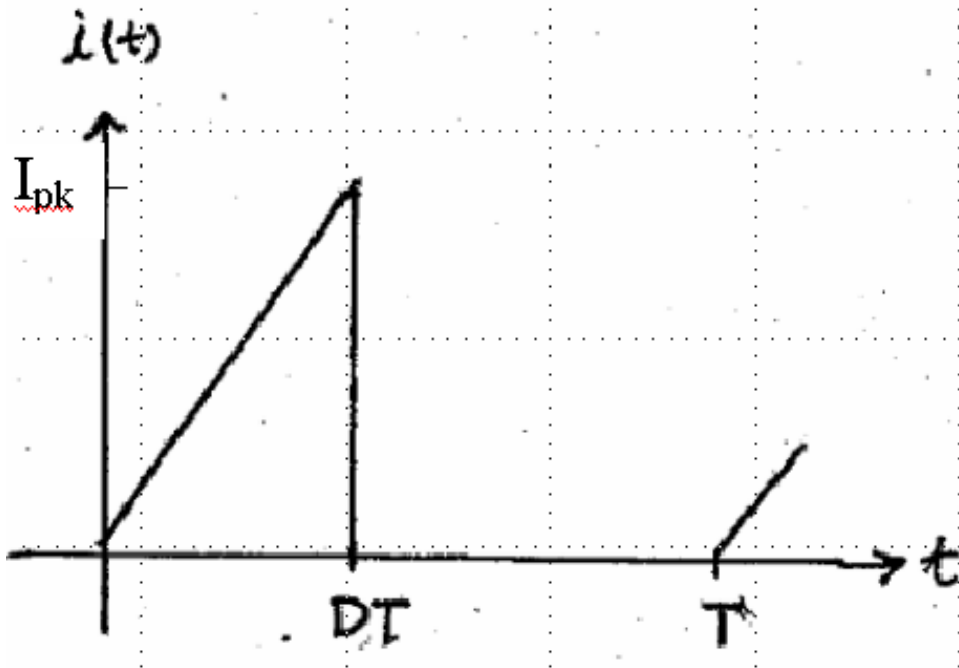
Pulsating with Ripple

- i.e. buck converter switch current
- We can use this result to get RMS value of buck diode current



$$\text{RMS} = I\sqrt{D} \sqrt{1 + \left(\frac{1}{3}\right) \left(\frac{\Delta i_{pp}}{2I}\right)^2}$$

Triangular



$$RMS = I_{pk} \sqrt{\frac{D}{3}}$$

Piecewise Calculation

- For I_1 , I_2 , etc. at different frequencies

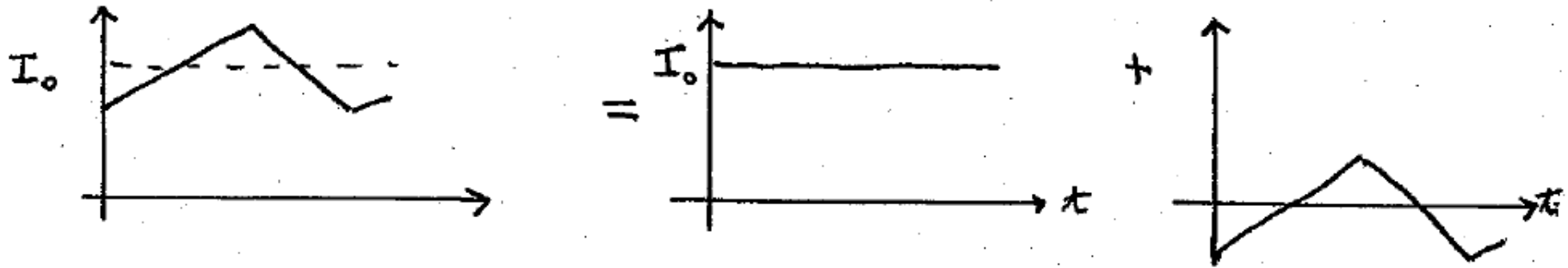
For $I = I_1 + I_2 + I_3 \dots$

then

$$I_{RMS} = \sqrt{I_{1,RMS}^2 + I_{2,RMS}^2 + \dots}$$

Piecewise Calculation --- Example

- What is RMS value of DC + ripple (shown before)?



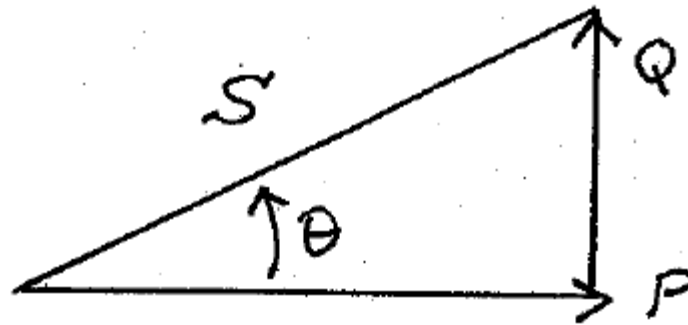
$$RMS = \sqrt{I_o^2 + \left(\frac{\Delta i_{pp}}{2\sqrt{3}}\right)^2} = I_o \sqrt{1 + \left(\frac{1}{3}\right)\left(\frac{\Delta i_{pp}}{2I_o}\right)^2}$$

Apparent, Real and Reactive Power

- “Power” has many shapes and forms
 - Real power (P, in Watts). This is the power that does work
 - Apparent power (S, in VA). This is the total “apparent” power seen by the source. It’s the product of V_{RMS} and I_{RMS}
 - Reactive power (Q, expressed in VAR)
 - Reactive power does not do real work
 - Instantaneous power $p(t) = v(t)i(t)$
 - Average power $\langle p(t) \rangle = \frac{1}{T} \int_0^T v(t)i(t)dt$

Vector Relationship Between P, Q and S

- P = real power (Watts)
- Q = reactive power (VAr)
- S = apparent power (VA)
- PF = power factor (unitless)

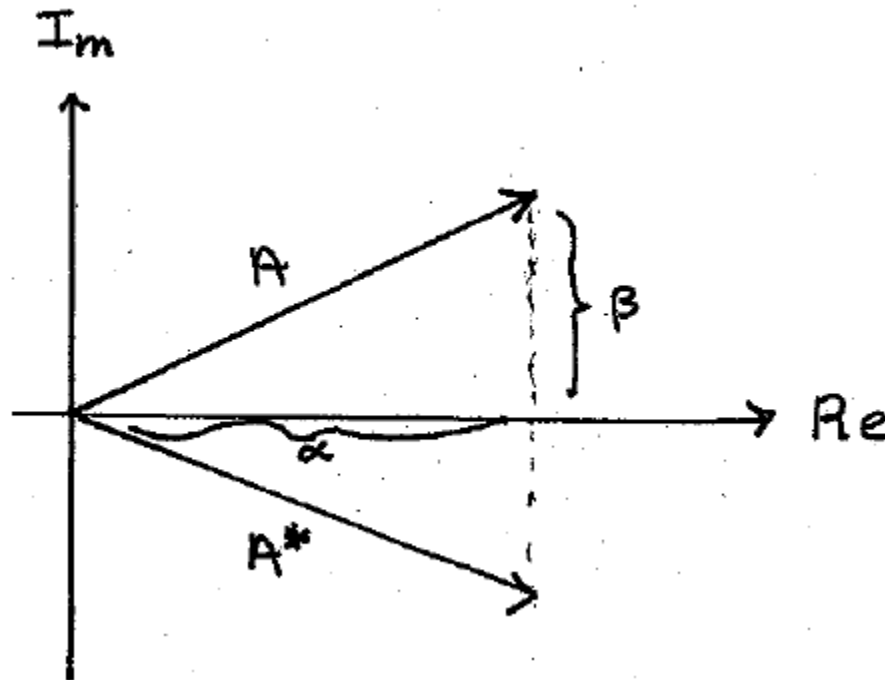


$$S = \sqrt{P^2 + Q^2}$$

$$PF = \frac{P}{S} = \cos(\theta)$$

Apparent Power from Voltage and Current

- We'll use the "Poynting vector" $S = V \times I^*$
- V = complex phase voltage, I^* = *complex conjugate* of phase current
- If $A = \alpha + j\beta$, then $A^* = \alpha - j\beta$
- $A \times A^* = \alpha^2 + \beta^2$



Apparent Power from Voltage and Current

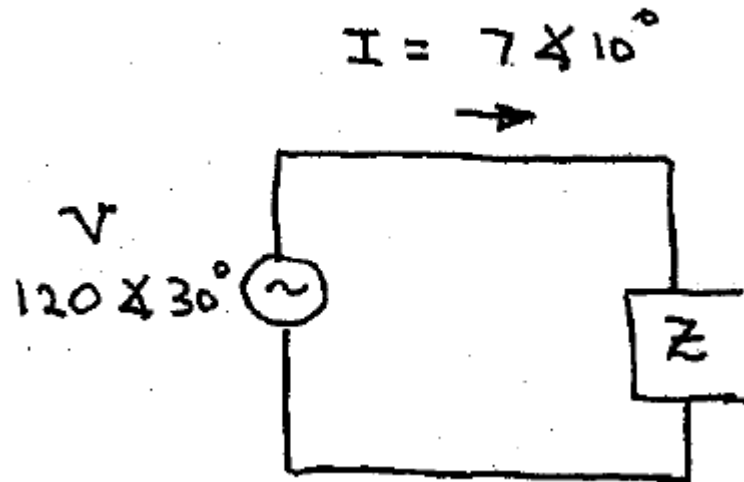
- Result:

$$P = |V||I| \cos(\theta)$$

$$Q = |V||I| \sin(\theta)$$

Example: Finding P, Q and S

- Let's find P, Q, S and Z given V and I

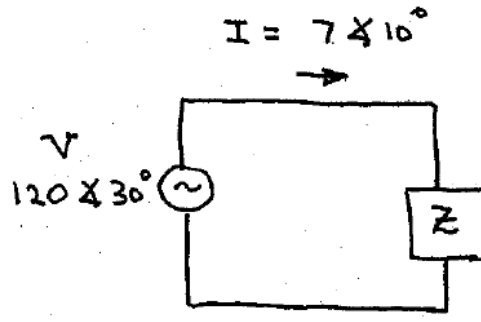


- Find apparent power S using Poynting vector

$$S = V \times I^* = (120 \angle 30^\circ)(7 \angle -10^\circ) = 840 \angle 20^\circ$$

$$S = 789 + j(287) \text{ VA}$$

Example: Finding P, Q and S



- Find real power P

$$P = VI \cos(\theta) = (120)(7) \cos(20^\circ) = 798 \text{ W}$$

- Find imaginary power Q

$$Q = VI \sin(\theta) = (120)(7) \sin(20^\circ) = 287 \text{ VAR}$$

- Find load impedance Z

$$Z = \frac{V}{I} = \frac{120 \angle 30^\circ}{7 \angle 10^\circ} = 17.14 \Omega \angle 20^\circ$$

Finding Load Current in Single Phase Systems

- We must know the current magnitude to properly size conductors. The NEC (National Electrical Code) gives guidance on sizing of conductors once we know the RMS conductor current
- Current magnitude results in I^2R loss and temperature rise in conductors

$$VI = VA = \frac{P}{PF}$$

$$I = \frac{P}{V \times PF}$$

Examples: Finding Load Current

- P=10 kVA, PF=1, 120V single phase

$$I = \frac{10000}{120} = 83.3A$$

- P=50 kVA, PF=0.9 lagging, 240V single phase

$$I = \frac{50000}{(240)(0.9)} = 231.5A$$

Sizing of Power Cables

- Large power cables are available in copper and aluminum
- Wire is sized by AWG (American Wire Gage) or thousands of circular mils (kcmils)
- The smallest wire used in power distribution is #14 AWG, typically a solid conductor with an outside diameter of 0.0641", or 64.1 mils
- The largest AWG is #4/0, with a diameter of approximately 0.522" for a 7-strand conductor

AWG

- Wire size is denoted by AWG (American Wire Gage)
- Wire diameter varies by a factor of 2 every 6 AWG
- #36 AWG is defined to be 0.005" (5 mil) diameter
- "Circular mils" is the diameter in mils, squared. (1 mil = 0.001")

To find wire diameter d based on AWG number:

$$\text{In inches: } d = (0.005") \times 92^{\frac{36-AWG}{39}}$$

$$\text{In millimeters: } d = (25.4) \times (0.005") \times 92^{\frac{36-AWG}{39}}$$

Circular Mils

- The area of larger conductors is expressed in circular mils. To find the area in cmils, square the diameter in mils by itself. For example, a #10 solid conductor with a diameter of 162 mils has an area of $(162 \text{ mils})^2 = 26244 \text{ cmils}$, or 26.2 kcmil

DC Resistance of Power Cables

- Resistance is a function of length, cross-sectional area, and electrical conductivity

$$R = \frac{l}{\sigma A}$$

- For copper, $\sigma \approx 5.8 \times 10^7 \text{ Ohm}^{-1}\text{m}^{-1}$ at room temperature.
For aluminum, $\sigma \approx 3.5 \times 10^7 \text{ Ohm}^{-1}\text{m}^{-1}$ at room temperature

Copper Temperature Coefficient of Resistivity

- Copper resistivity goes up as temperature goes up
- So, the total resistance of a piece of wire goes up as it heats up
- Temperature coefficient of resistivity is about +0.004/°C, or about 0.4% per degree C
- Resistance goes up as temperature goes up.

$$R_{T_2} = R_{25} [1 + \alpha(T_2 - 298)]$$

- R_{25} = resistance at 25C (298K), α = temperature coefficient ≈ 0.00385 for copper and ≈ 0.00395 for aluminum; rule of thumb for copper is +0.4% per degree C

Issues With Aluminum Wire

- Aluminum has higher resistivity than copper (about 70% higher), with resistivity $\rho = 2.9 \times 10^{-8} \Omega\text{-m}$
- A 15-amp branch circuit made with #14 copper would require #12 aluminum, by the NEC
- Similar TC to copper
- Aluminum is much lighter than copper (about 33% of the weight per unit volume)
- Often used in power transmission (good ratio of resistivity to weight)
- Corrosion when joining aluminum to copper lugs; differing thermal expansion

Power Cables and Cable Impedance

- Power cables have a finite resistance (due to the resistivity of the conductor) and a finite inductance (due to the geometry of the cable)
- Cable impedance results in voltage drops, power loss, and heating in the cable

AC Cable Resistance

- DC resistance of a piece of wire:

$$R = \frac{l}{\sigma A}$$

- A = cross sectional area of wire, l = length, and σ = electrical conductivity
- However, AC resistance is higher than DC resistance, due to skin effect and proximity effect
- Wire resistance can be found in a “wire chart” for an isolated wire (i.e. in air, no metallic conduit, no other current-carrying wires nearby)

Copper Wire Data

Kassakian, Sciacca & Vergara
 Principles of Power Electronics
 582 Chapter 20 Magnetic Components

Addison-Wesley
 1991

Table 20.1 Copper Wire Data

AWG SIZE	DIAMETER (MM)	Ω/KM (75°C)	KG/KM	TURNS/CM ²
0	8.25	0.392	475	
1	7.35	0.494	377	
2	6.54	0.624	299	
3	5.83	0.786	237	
4	5.19	0.991	188	
5	4.62	1.25	149	
6	4.12	1.58	118	
7	3.67	1.99	93.8	
8	3.26	2.51	74.4	
9	2.91	3.16	59.0	
10	2.59	3.99	46.8	14
11	2.31	5.03	37.1	17
12	2.05	6.34	29.4	22
13	1.83	7.99	23.3	27
14	1.63	10.1	18.5	34
15	1.45	12.7	14.7	40
16	1.29	16.0	11.6	51
17	1.15	20.2	9.23	63
18	1.02	25.5	7.32	79
19	0.912	32.1	5.80	98
20	0.812	40.5	4.60	123
21	0.723	51.1	3.65	153
22	0.644	64.4	2.89	192
23	0.573	81.2	2.30	237
24	0.511	102	1.82	293
25	0.455	129	1.44	364
26	0.405	163	1.15	454
27	0.361	205	1.10	575
28	0.321	259	1.39	710
29	0.286	327	1.75	871
30	0.255	412	2.21	1090

1 LB
 350 FEET

Copper Wire Data

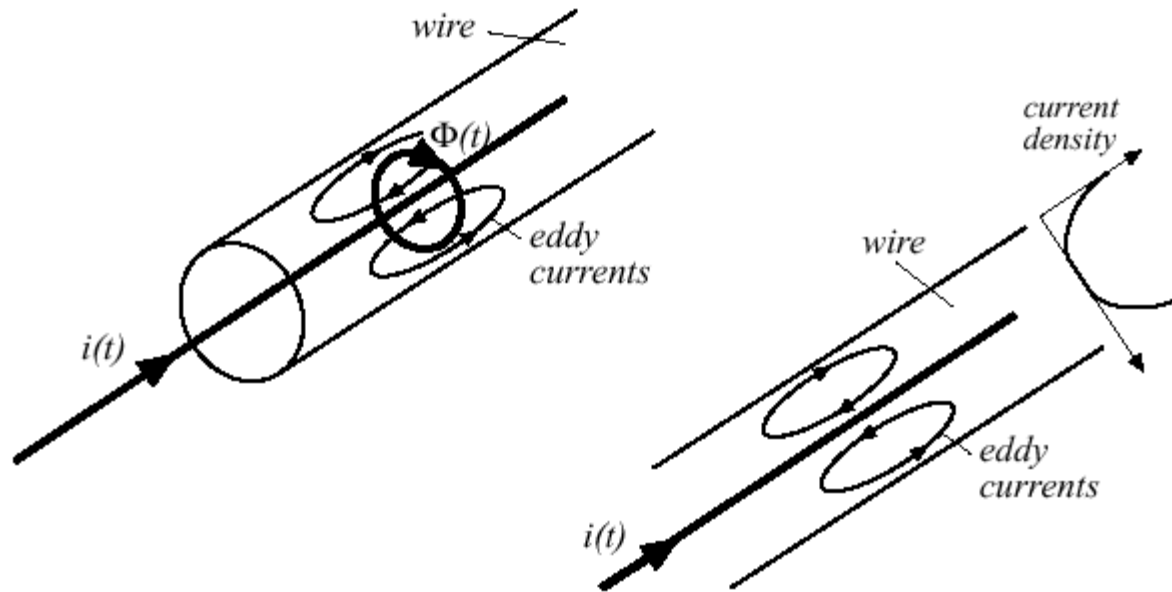
TABLE 11.1
Standard Wire Sizes and Current Capabilities

AWG Number	Diameter (mm)	Cross-Sectional Area (mm ²)	Resistance, mΩ/m at 25°C	Current Capacity, 500 A/cm ²	Current Capacity, 100 A/cm ²
4	5.189	21.15	0.8314	105.8	21.15
6	4.115	13.30	1.322	66.51	13.30
8	3.264	8.366	2.102	41.83	8.366
10	2.588	5.261	3.343	26.31	5.261
12	2.053	3.309	5.315	16.54	3.309
14	1.628	2.081	8.451	10.40	2.081
16	1.291	1.309	13.44	6.543	1.309
18	1.024	0.8230	21.36	4.115	0.8230
20	0.8118	0.5176	33.96	2.588	0.5176
22	0.6438	0.3255	54.00	1.628	0.3255
24	0.5106	0.2047	85.89	1.024	0.2047
26	0.4049	0.1288	136.5	0.6438	0.1288
28	0.3211	0.08098	217.1	0.4049	0.08098
30	0.2546	0.05093	345.1	0.2546	0.05093
32	0.2019	0.03203	549.3	0.1601	0.03203
34	0.1601	0.02014	873.3	0.1007	0.02014
36	0.127000	0.0126677	1389.	0.06334	0.0126677
38	0.1007	0.007967	2208.	0.03983	0.007967
40	0.07987	0.005010	3510.	0.02505	0.005010

Reference: P. Krein, *Elements of Power Electronics*, 3d edition

Causes of Skin Effect

- Self-field of wire causes current to crowd on the surface of the wire, raising the AC resistance
- Method:
 - Current $i(t) \Rightarrow$ changing magnetic flux density $B(t) \Rightarrow$ reaction currents



Reference: <http://ece-www.colorado.edu/~pwrelect/book/slides/Ch12slide.pdf>

Effects of Skin Effect

- For high frequencies, current is concentrated in a layer approximately one skin depth δ thick
- Skin depth varies with frequency

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

- σ = electrical conductivity = $5.9 \times 10^7 \text{ } \Omega^{-1} \text{ m}^{-1}$ for copper at 300K
- μ = magnetic permeability of material = $4\pi \times 10^{-7} \text{ H/m}$ in free space

<i>Skin depth in copper at 300K</i>		
cond	5.90E+07	
mu	1.26E-06	
<i>f</i>	<i>skin depth (meter)</i>	
1	6.55E-02	
10	2.07E-02	
100	6.55E-03	
1000	2.07E-03	
1.00E+04	6.55E-04	
1.00E+05	2.07E-04	
1.00E+06	6.55E-05	

Skin Effect --- Increase in Wire Resistance

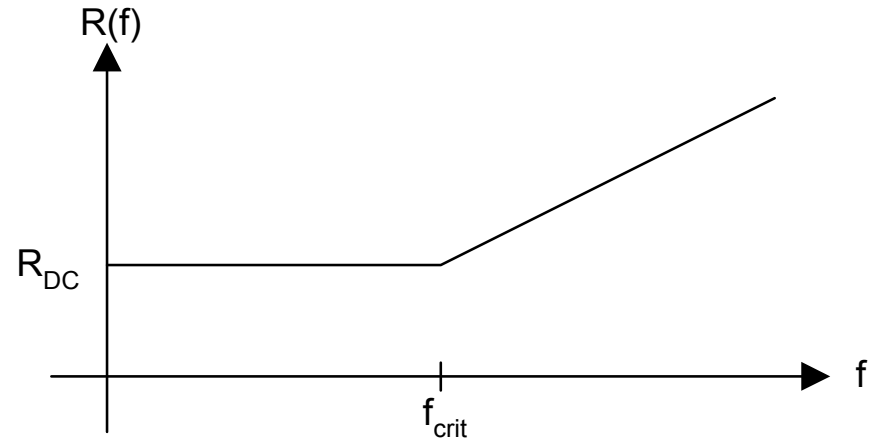
- For high frequencies, resistance of wire increases

- DC resistance of wire:
$$R_{DC} = \frac{l}{\sigma(\pi r_w^2)}$$

- For frequencies above critical frequency where skin depth equals wire radius r_w :

$$R_{AC} = \frac{l}{\sigma(2\pi r_w \delta)} = R_{DC} \left(\frac{2\delta}{r_w} \right)$$

$$f_{crit} = \frac{1}{\pi r_w^2 \mu \sigma}$$

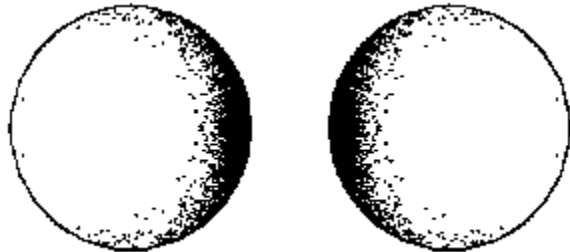


- Result: for high frequency operation, don't bother making wire radius $> \delta$
 - Skin depth in copper at 60 Hz is approximately 8 mm

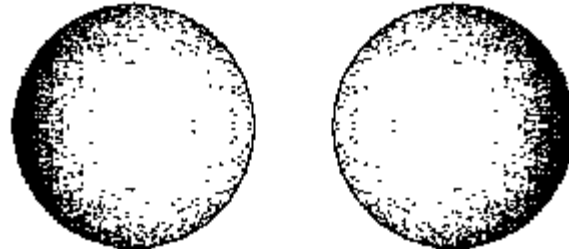
Proximity Effect

- In multiple-layer windings in inductors and transformers, the proximity effect can also greatly increase the winding resistance
- Field from one wire affects the current profile in another

Currents in opposite direction



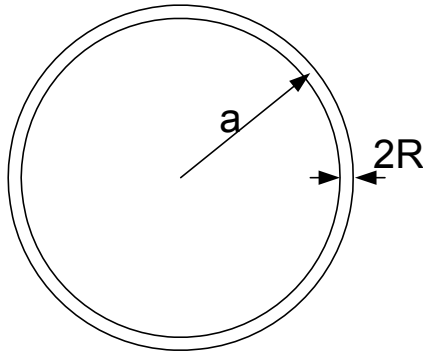
Currents in same direction



Cable Inductance

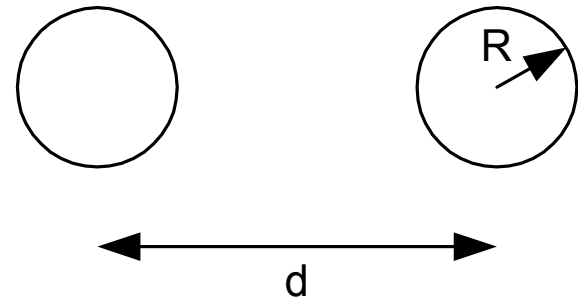
- Inductance is defined by the geometry of the wire loop
- Included in the calculation is wire radius, and loop length and shape

Circular loop of wire



$$L = \mu_o a \left[\ln \left(\frac{8a}{R} \right) - 1.75 \right]$$

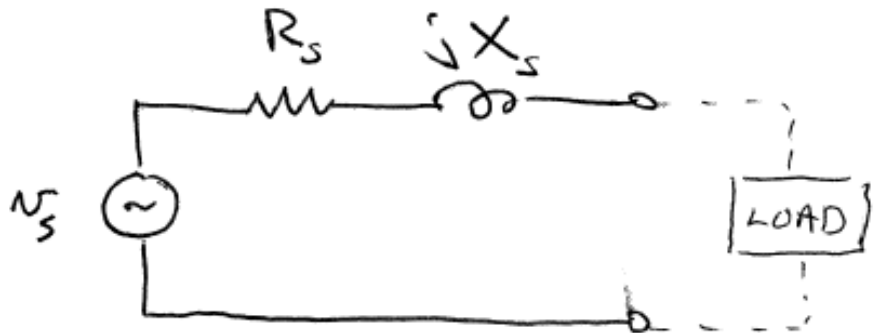
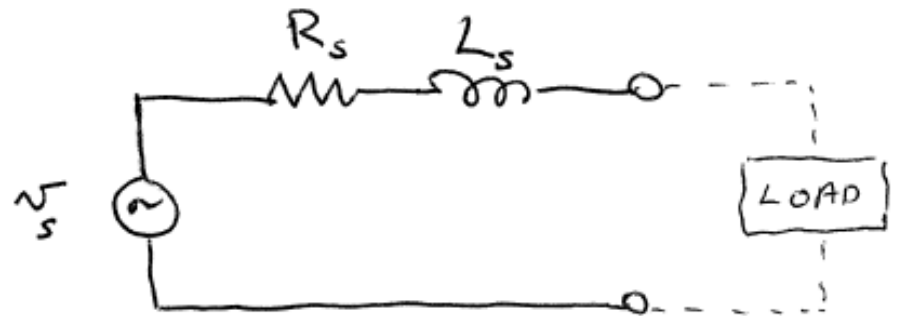
Parallel-wire line



$$L = \frac{\mu_o l}{\pi} \ln \left[\frac{d}{R} + \frac{1}{4} - \frac{d}{l} \right]$$

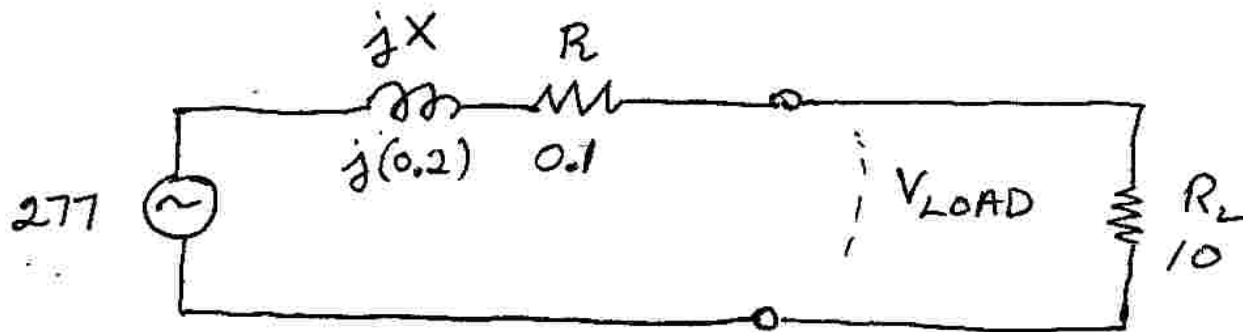
AC Service with Real-World Impedance

- All electrical services have finite impedance; shown here the impedance is $R_s + j\omega L_s$
- These types of power distribution drawings rarely show inductance explicitly; rather the inductive reactance jX_s is used.
- L_s may be the sum of wiring inductance, transformer leakage, etc.



Example: Finding Load Voltage and Current

- Find load voltage and load current
- Note that line resistance is 0.1Ω and line reactance is 0.2Ω



$$I_L = \frac{277}{0.2j + 0.1 + 10} = 27.4 - 0.54j$$

$$V_{LOAD} = I_L R_L = 274 - 5.4j$$

- Note voltage drop across line impedance of a few Volts

National Elec. Code Impedance Estimates

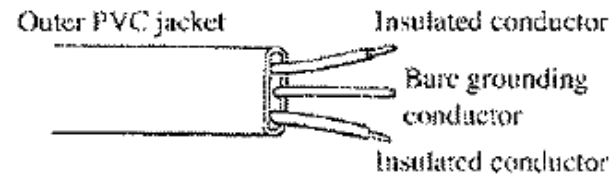
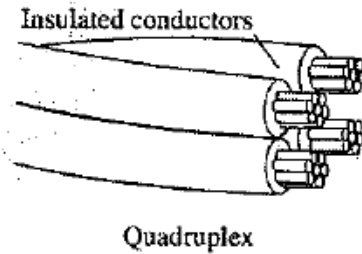
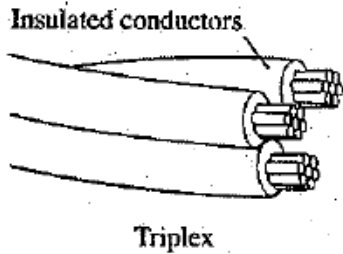
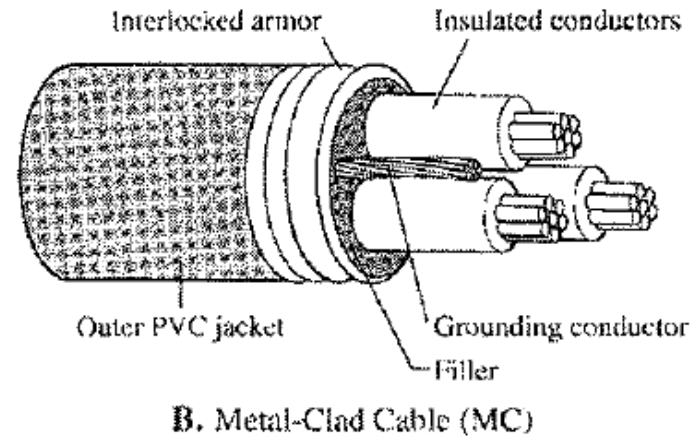
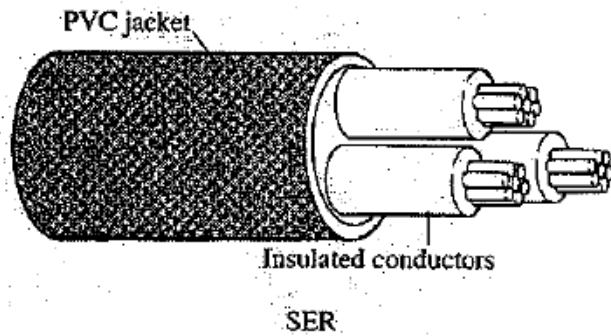
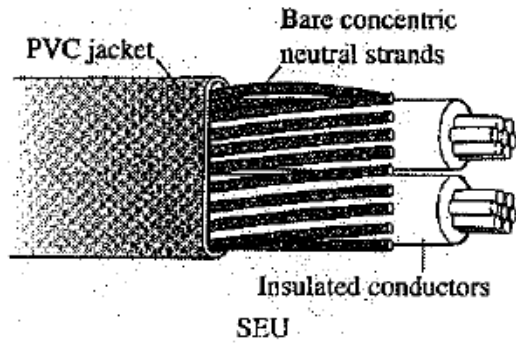
- Note conduit type affects resistance and reactance

TABLE 16-1

Table 9 Alternating-Current Resistance and Reactance for 600-Volt Cables, 3-Phase, 60 Hz, 75°C (167°F)—
Three Single Conductors in Conduit

Size (AWG or kcmil)	Ohms to Neutral per Kilometer Ohms to Neutral per 1000 Feet															Size (AWG or kcmil)
	X_L (Reactance) for All Wires		Alternating-Current Resistance for Uncoated Copper Wires			Alternating-Current Resistance for Aluminum Wires			Effective Z at 0.85 PF for Uncoated Copper Wires			Effective Z at 0.85 PF for Aluminum Wires				
	PVC, Aluminum Conduits	Steel Conduit	PVC Conduit	Aluminum Conduit	Steel Conduit	PVC Conduit	Aluminum Conduit	Steel Conduit	PVC Conduit	Aluminum Conduit	Steel Conduit	PVC Conduit	Aluminum Conduit	Steel Conduit		
14	0.190 0.058	0.240 0.073	10.2 3.1	10.2 3.1	10.2 3.1	— —	— —	— —	8.9 2.7	8.9 2.7	8.9 2.7	— —	— —	— —	14	
12	0.177 0.054	0.223 0.058	6.6 2.0	6.6 2.0	6.6 2.0	10.5 3.2	10.5 3.2	10.5 3.2	5.6 1.7	5.6 1.7	5.6 1.7	9.2 2.8	9.2 2.8	9.2 2.8	12	
10	0.164 0.050	0.207 0.063	3.9 1.2	3.9 1.2	3.9 1.2	6.6 2.0	6.6 2.0	6.6 2.0	3.6 1.1	3.6 1.1	3.6 1.1	5.9 1.8	5.9 1.8	5.9 1.8	10	
8	0.171 0.052	0.213 0.065	2.56 0.78	2.56 0.78	2.56 0.78	4.3 1.3	4.3 1.3	4.3 1.3	2.26 0.69	2.26 0.69	2.30 0.70	3.6 1.1	3.6 1.1	3.6 1.1	8	
6	0.167 0.051	0.210 0.064	1.61 0.49	1.61 0.49	1.61 0.49	2.66 0.81	2.66 0.81	2.66 0.81	1.44 0.44	1.48 0.45	1.48 0.45	2.33 0.71	2.36 0.72	2.36 0.72	6	
4	0.157 0.048	0.197 0.060	1.02 0.31	1.02 0.31	1.02 0.31	1.67 0.51	1.67 0.51	1.67 0.51	0.95 0.29	0.95 0.29	0.98 0.30	1.51 0.46	1.51 0.46	1.51 0.46	4	
3	0.154 0.047	0.194 0.059	0.82 0.25	0.82 0.25	0.82 0.25	1.31 0.40	1.35 0.41	1.31 0.40	0.75 0.23	0.79 0.24					3	

Power Cable Types



C. Nonmetallic Sheathed Cable (NM)

USE

Power Cable Types

TABLE 6-1
Cable Insulation Types

THW-2	<u>T</u> hermoplastic insulation (usually PVC), <u>H</u> eat resistant (90°C rating), suitable for <u>W</u> et locations.
THWN-2	Same as THW except <u>N</u> ylon jacket over reduced insulation thickness. Also rated THHN.
THHN	<u>T</u> hermoplastic insulation (usually PVC), <u>H</u> igh <u>H</u> eat resistant (90°C rating), dry locations only, <u>N</u> ylon jacket. Also rated THWN.
XHHW-2	Cross-linked polyethylene insulation (<u>X</u>), <u>H</u> igh <u>H</u> eat resistant (90°C rating), for wet and dry locations.
RHH	<u>R</u> ubber insulation. Most manufacturers use cross-linked polyethylene because it has the same properties as rubber. <u>H</u> igh <u>H</u> eat resistant (90°C rating), for dry locations only.
RHW-2	<u>R</u> ubber insulation (cross-linked polyethylene), <u>H</u> eat resistant (90°C rating), suitable for <u>W</u> et locations.
USE-2	<u>U</u> nderground <u>S</u> ervice <u>E</u> ntrance. Most utilize XLP for 90°C in direct burial applications. Product is usually triple rated: RHH-RHW-USE.
NM-B	<u>N</u> on <u>M</u> etallic sheathed cable. The " <u>B</u> " denotes that individual conductor insulation is rated 90°C; however, ampacity is limited to that of a 60°C conductor. Thermoplastic (PVC) conductor insulation, nylon jacketed, with overall PVC cable jacket.
SEU	<u>S</u> ervice <u>E</u> ntrance Cable, <u>U</u> narmored. Usually type XHHW insulated conductors with overall PVC jacket. As such, the cable is rated for 90°C dry, 75°C wet locations.
SER	<u>S</u> ervice <u>E</u> ntrance Cable, <u>R</u> ound. Same material construction as SEU but round construction.

Source: Southwire Power Cable Manual and Product Catalog. © Southwire Company.

National Elec. Code Ampacity Estimates

Table 310.16 Allowable Ampacities of Insulated Conductors Rated 0 Through 2000 Volts, 60°C Through 90°C (140°F Through 194°F), Not More Than Three Current-Carrying Conductors in Raceway, Cable, or Earth (Directly Buried), Based on Ambient Temperature of 30°C (86°F)

9

Size AWG or kcmil	Temperature Rating of Conductor (See Table 310.13.)					
	60°C (140°F)	75°C (167°F)	90°C (194°F)	60°C (140°F)	75°C (167°F)	90°C (194°F)
	Types TW, UF	Types RHW, THHW, THW, THWN, XHHW, USE, ZW	Types TBS, SA, SIS FEP, FEPB, MI, RHH, RHW-2, THHN, THHW, THW-2, THWN-2, USE-2, XHH, XHHW, XHHW-2 ZW-2	Types TW, UF	Types RHW, THHW, THW, THWN, XHHW, USE	Types TBS, SA, SIS, THHN, THHW, THW-2, THWN-2, RHH, RHW-2, USE-2, XHH, XHHW, XHHW-2, ZW-2
	COPPER			ALUMINUM OR COPPER-CLAD ALUMINUM		
18	—	—	14	—	—	—
16	—	—	18	—	—	—
14*	20	20	25	—	—	—
12*	25	25	30	20	20	25
(a) → 10*	30	35	40	25	30	35
8	40	50	55	30	40	45
6	55	65	75	40	50	60
4	70	85 (b)	95	55	65	75
3	85	100	110	65	75	85
2	95	115	130	75	90	100
1	110	130	150	85	100	115
1/0	125	150	170	100	120	135
2/0	145	175	195	115	135	150
3/0	165	200	225	130	155	175
4/0	195	230	260	150	180	205
250	215	255	290	170	205	230
300	240	285	320	190	230	255
350	260	310	350	210	250	280
400	280	335	380	225	270	305

National Elec. Code Ampacity Estimates

CORRECTION FACTORS

Ambient Temp. (°C)	For ambient temperatures other than 30°C (86°F), multiply the allowable ampacities shown above by the appropriate factor shown below					
21-25	1.08	1.05	1.04	1.08	1.05	1.04
26-30	1.00	1.00	1.00	1.00	1.00	1.00
31-35	0.91	0.94	0.96	0.91	0.94	0.96
36-40	0.82	0.88	0.91	0.82	0.88	0.91
41-45	0.71	0.82	0.87	0.71	0.82	0.87
46-50	0.58	0.75	0.82	0.58	0.75	0.82
51-55	0.41	0.67	0.76	0.41	0.67	0.76
56-60	—	0.58	0.71	—	0.58	0.71
61-70	—	0.33	0.58	—	0.33	0.58
71-80	—	—	0.41	—	—	0.41

*See 240.4(D).

“Single-Line Drawings”

- Also called “one-line” drawing
- Shorthand to describe power distribution systems
- Symbols include power busses, transmission lines, disconnect switches, air and oil-filled circuit breakers, transformers, motors, generators, etc.
- Shown here is a “radial” power distribution system

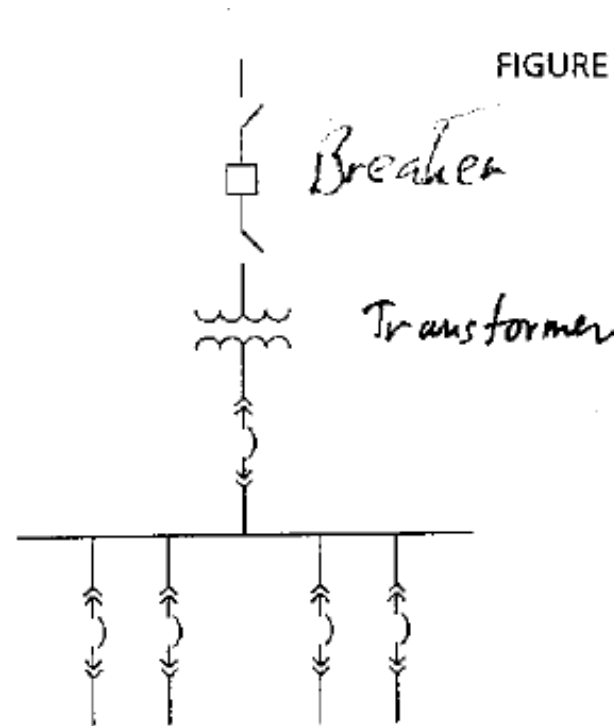


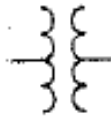
FIGURE 1-3 Radial System

Single-Line Drawing Symbols

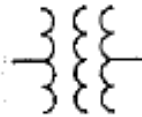
- There are lots more, but these are some of the important ones



ROTATING MACHINE



TWO WINDING POWER TRANSFORMER



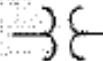
THREE WINDING POWER TRANSFORMER



FUSE



CURRENT TRANSFORMER



POTENTIAL TRANSFORMER



AIR CIRCUIT BREAKER



OIL CIRCUIT BREAKER



THREE-PHASE WYE, NEUTRAL UNGROUNDED

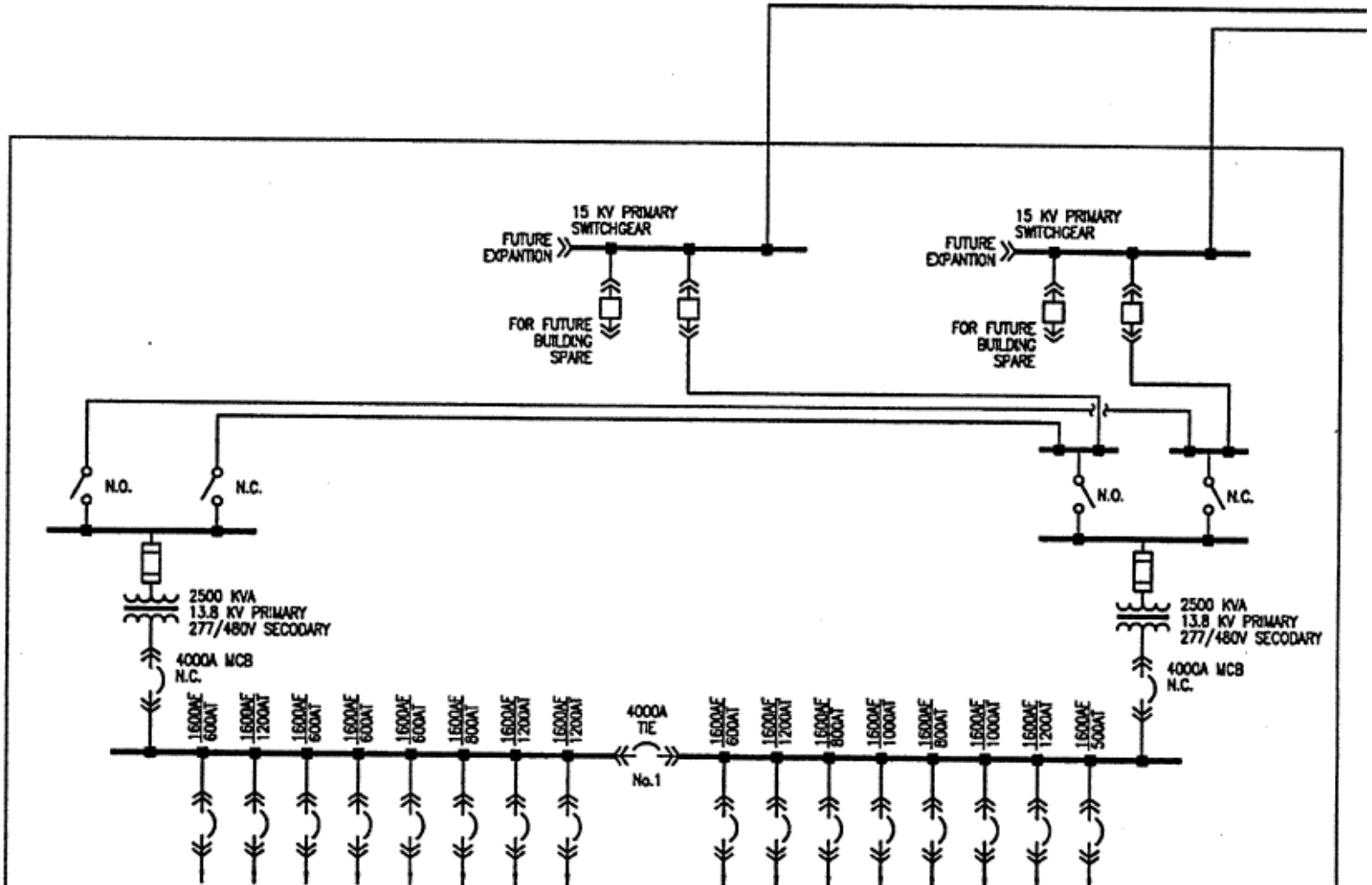


THREE-PHASE WYE, GROUNDED NEUTRAL

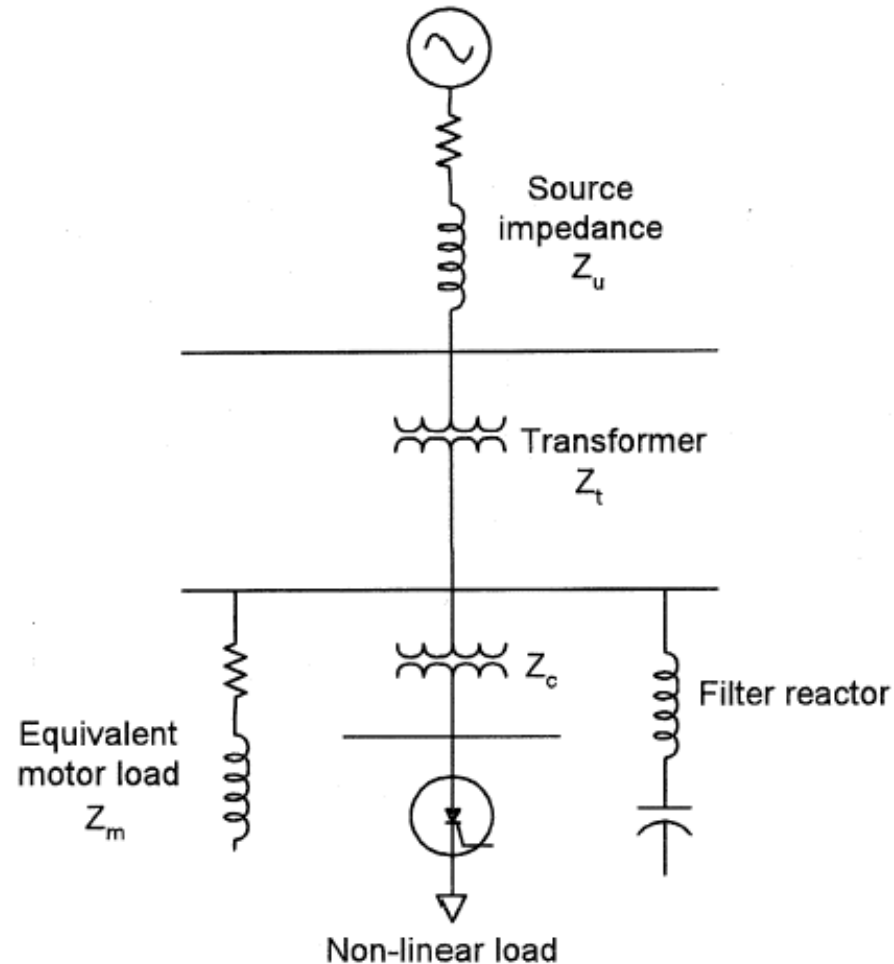


THREE-PHASE DELTA

Typical Single-Line Drawing



Typical Single-Line Drawing



Reference: J. C. Das, "Passive Filters --- Potentialities and Limitations," *IEEE Transactions on Industry Applications*, vol. 40, no. 1, January/February, 2004, pp. 232-241

Power Factor and Power Factor Correction

- Power factor is an important concept in power circuits, power electronics, and electric motors
- Power factor is a measure of how easily you deliver real power to a load

Power Factor

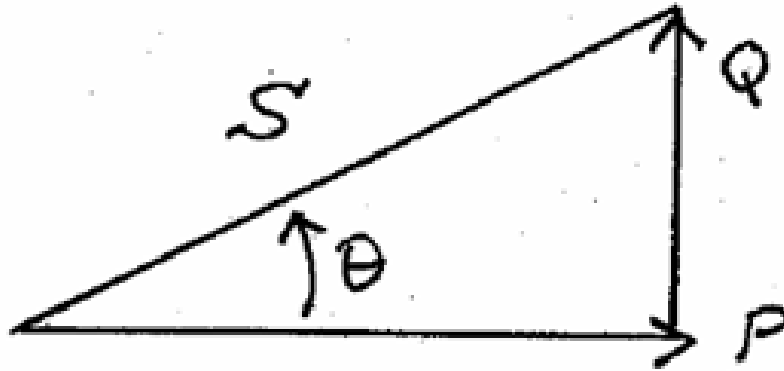
- Ratio of delivered power to the product of RMS voltage and RMS current

$$PF = \frac{\langle P \rangle}{V_{RMS} I_{RMS}}$$

- Power factor always ≤ 1
- With pure sine wave and resistive load, $PF = 1$
- With pure sine wave and purely reactive load, $PF = 0$
- Whenever $PF < 1$ the circuit carries currents or voltages that do not perform useful work
- The more “spikey” a waveform is the worse is its PF
 - Diode rectifiers have poor power factor
- Power factor can be helped by “power factor correction”

Example: Power Factor Calculation

- A 3-phase load consumes 100 kW and 50 kVAr
- Find P, Q, S and power factor PF



$$P = 100 \text{ kW} = 10^5$$

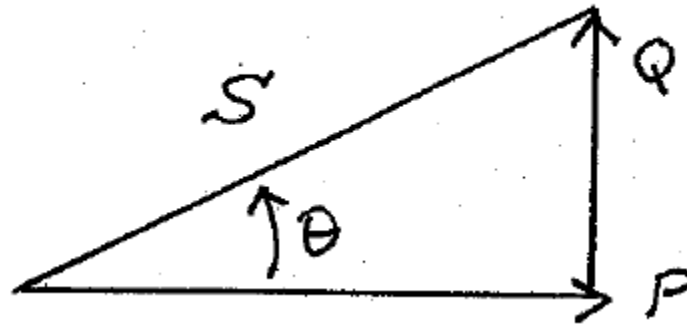
$$Q = 50 \text{ kVAr}$$

$$S = \sqrt{(10^5)^2 + (5 \times 10^4)^2} = 111.8 \text{ kVA}$$

$$PF = \frac{P}{S} = \frac{100}{111.8} = 0.894$$

Example: Another PF Calculation

- A load consumes 500 kW at 0.85 PF lagging. Find apparent power, reactive power, and power factor angle



$$P = 500 \text{ kW} = 5 \times 10^5$$

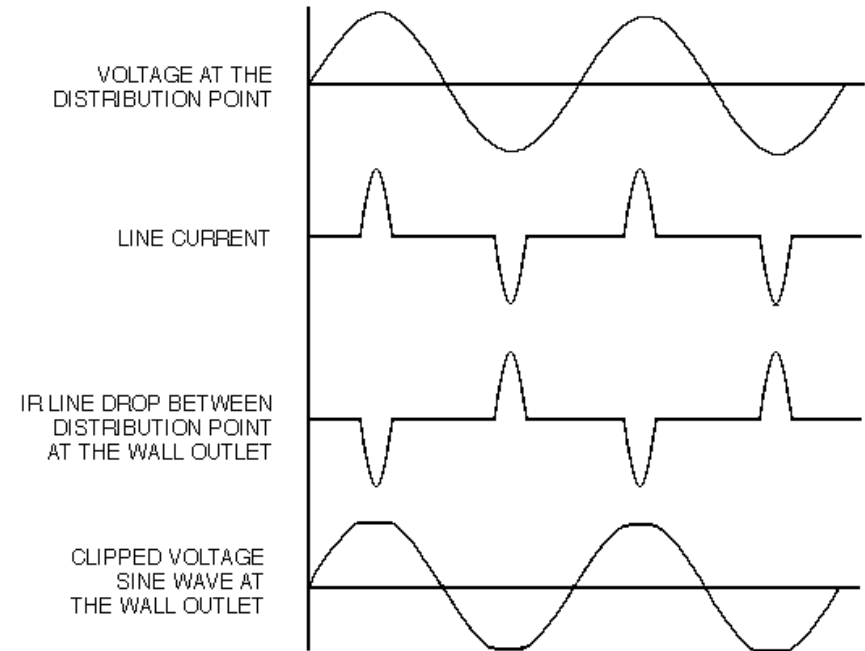
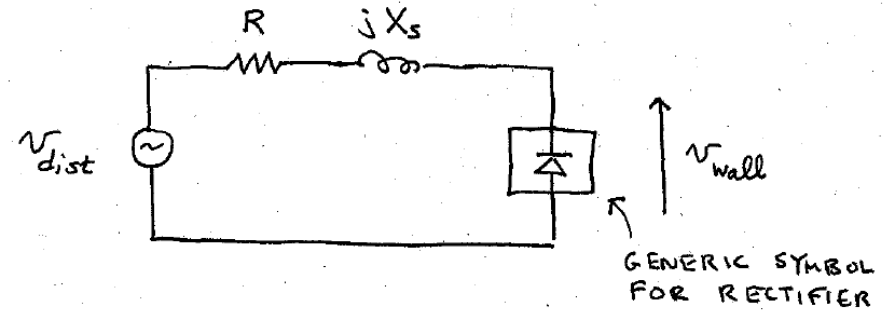
$$PF = \frac{P}{S} \Rightarrow S = \frac{P}{PF} = \frac{5 \times 10^5}{0.85} = 588 \text{ kVA}$$

$$P^2 + Q^2 = S^2 \Rightarrow Q = \sqrt{S^2 - P^2} = \sqrt{588^2 - 500^2} = 310 \text{ kVA}_r$$

$$\theta = \cos^{-1}(PF) = \cos^{-1}(0.85) = -31.8^\circ$$

Causes of Low Power Factor --- Nonlinear Load

- Nonlinear loads include:
 - Variable-speed drives
 - Frequency converters
 - Uninterruptible power supplies (UPS)
 - Saturated magnetic circuits
 - Dimmer switches
 - Televisions
 - Fluorescent lamps
 - Welding sets
 - Arc furnaces
 - Semiconductors
 - Battery chargers



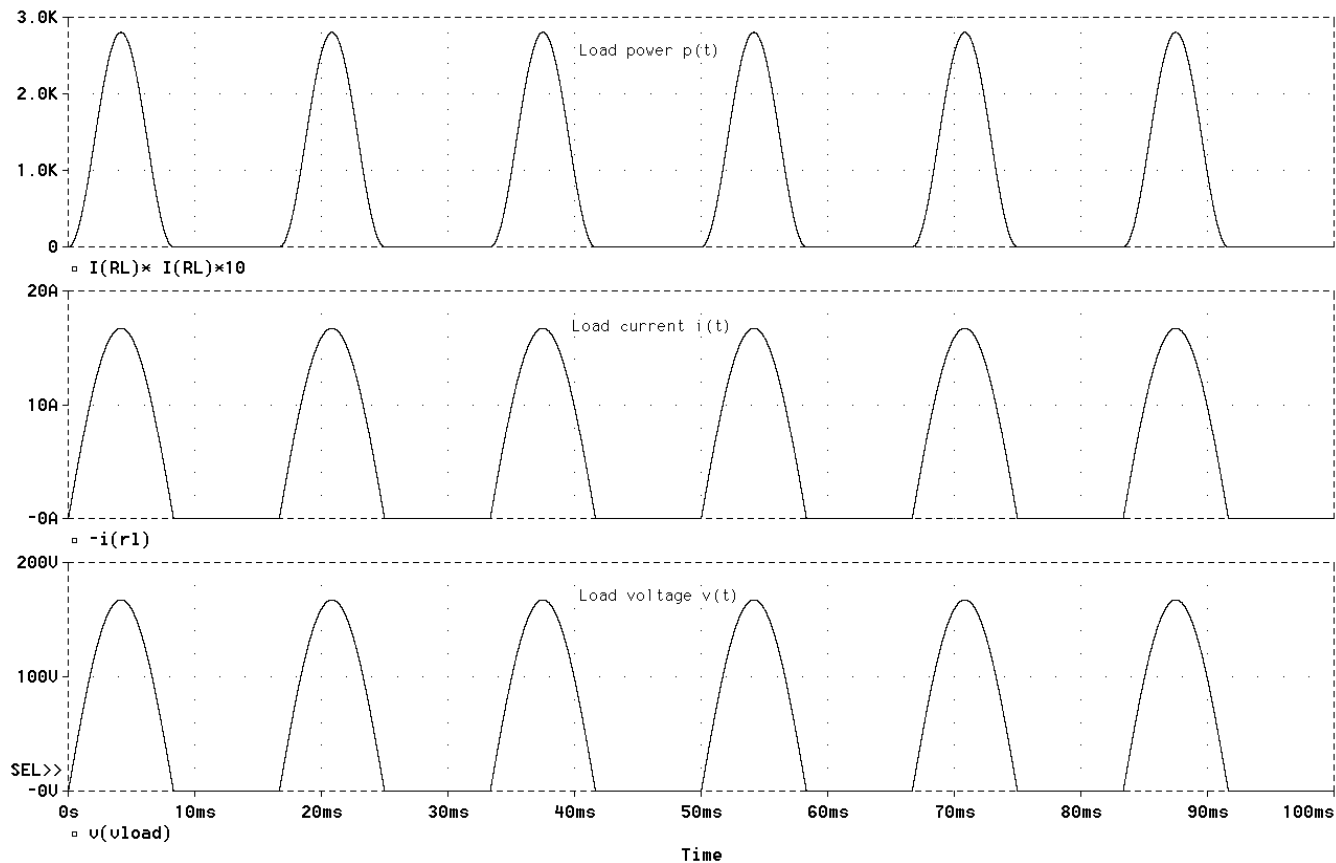
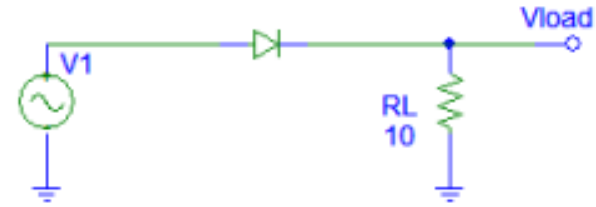
Further Comment on Power Factor

- Power factor (k_p) can be broken up into 2 pieces:
- Displacement power factor k_θ , due to phase shift between 1st harmonic of current and input voltage
- Distortion power factor (k_d) due to harmonics
- Total power factor is product of these 2

$$k_p = k_d k_\theta$$

HWR with Resistive Load

- Note that power is delivered in periodic blips; line current has harmonics so $PF < 1$



HWR with Resistive Load --- Power Factor

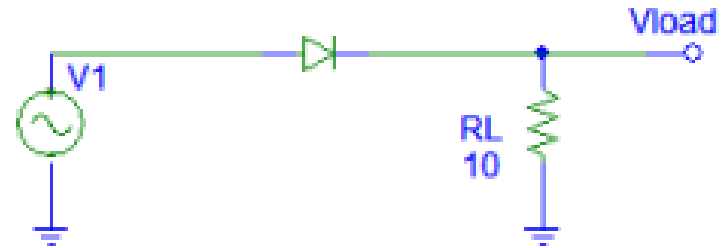
- We can calculate P.F. by inspection:

$$v_{in,RMS} = \frac{V_{pk}}{\sqrt{2}}$$

$$i_{in,RMS} = \left(\frac{V_{pk}}{\sqrt{2}} \right) \left(\frac{1}{R_L} \right) \left(\frac{1}{\sqrt{2}} \right) = \frac{V_{pk}}{2R_L}$$

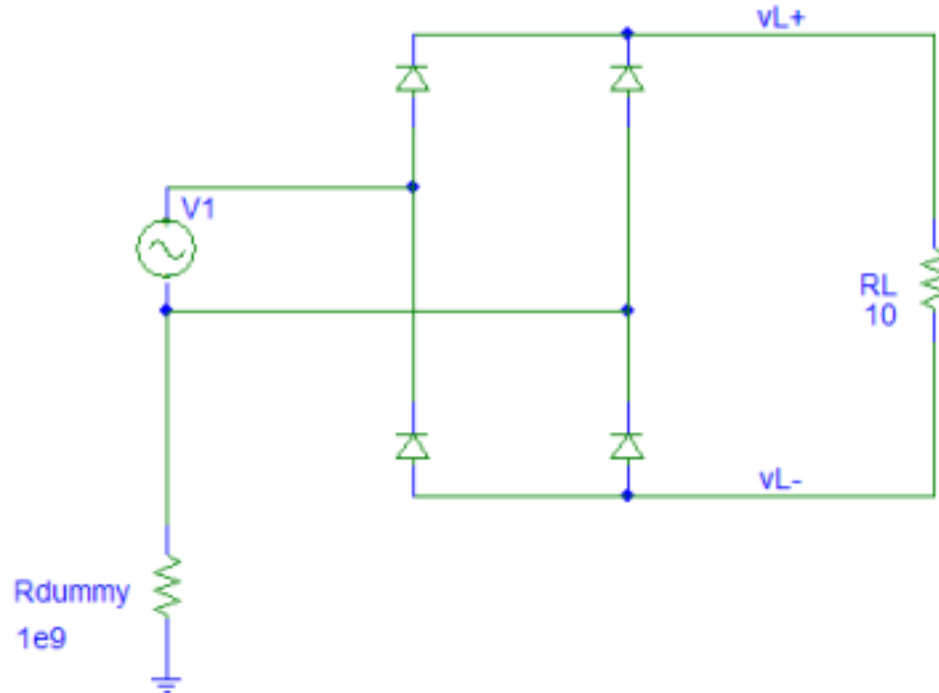
$$\langle p_L \rangle = i_{in,RMS}^2 R_L = \left(\frac{V_{pk}}{2R_L} \right)^2 R_L = \frac{V_{pk}^2}{4R_L}$$

$$P.F. = \frac{\langle p_L \rangle}{v_{in,RMS} i_{in,RMS}} = \frac{\left(\frac{V_{pk}^2}{4R_L} \right)}{\left(\frac{V_{pk}}{\sqrt{2}} \right) \left(\frac{V_{pk}}{2R_L} \right)} = \frac{\sqrt{2}}{2} = 0.707$$

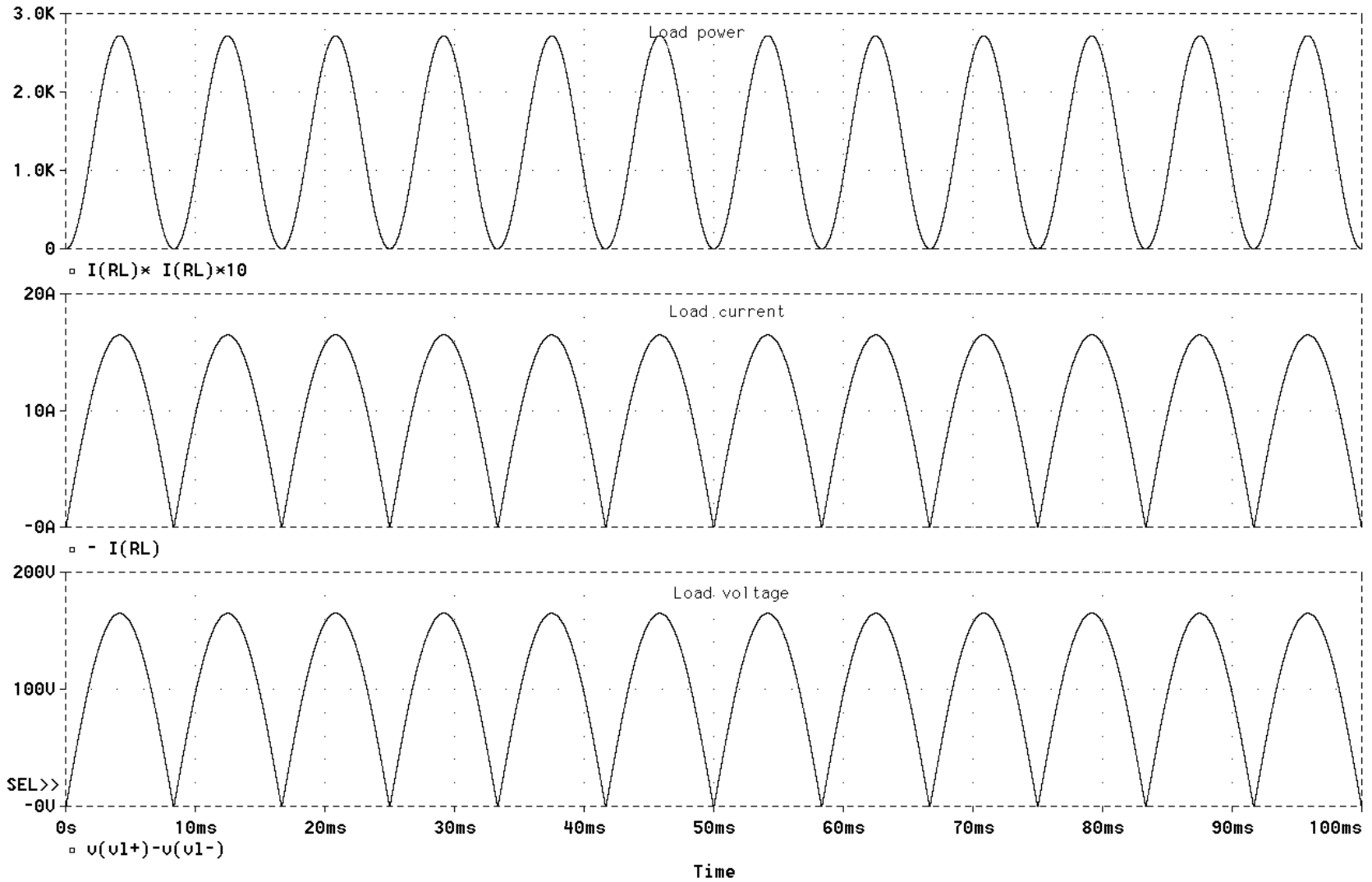


FWR with Resistive Load --- PSPICE Simulation

- We need dummy resistor to keep PSPICE happy



FWR with Resistive Load



FWR with Resistive Load --- Power Factor

$$v_{in,RMS} = \frac{V_{pk}}{\sqrt{2}}$$

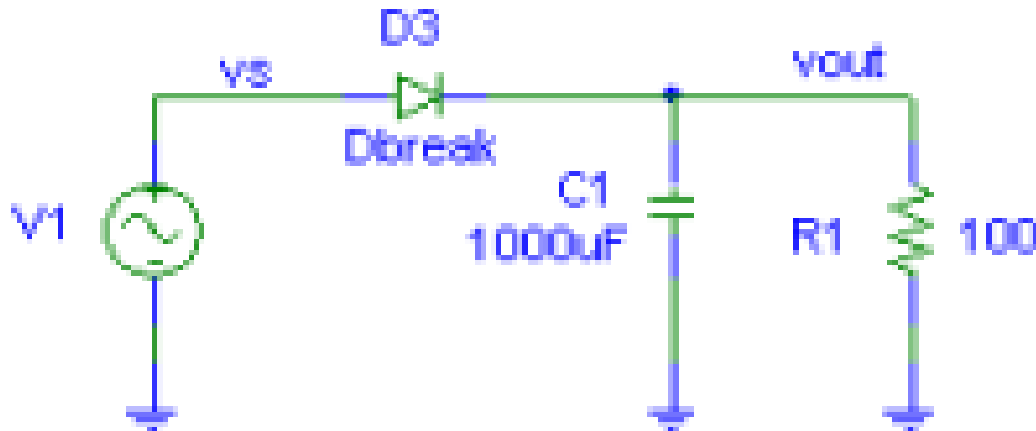
$$i_{in,RMS} = \left(\frac{V_{pk}}{\sqrt{2}} \right) \left(\frac{1}{R_L} \right) = \frac{V_{pk}}{\sqrt{2}R_L}$$

$$\langle p_L \rangle = i_{in,RMS}^2 R_L = \left(\frac{V_{pk}}{\sqrt{2}R_L} \right)^2 R_L = \frac{V_{pk}^2}{2R_L}$$

$$P.F. = \frac{\langle p_L \rangle}{v_{in,RMS} i_{in,RMS}} = \frac{\left(\frac{V_{pk}^2}{2R_L} \right)}{\left(\frac{V_{pk}}{\sqrt{2}} \right) \left(\frac{V_{pk}}{\sqrt{2}R_L} \right)} = \frac{2}{2} = 1.0$$

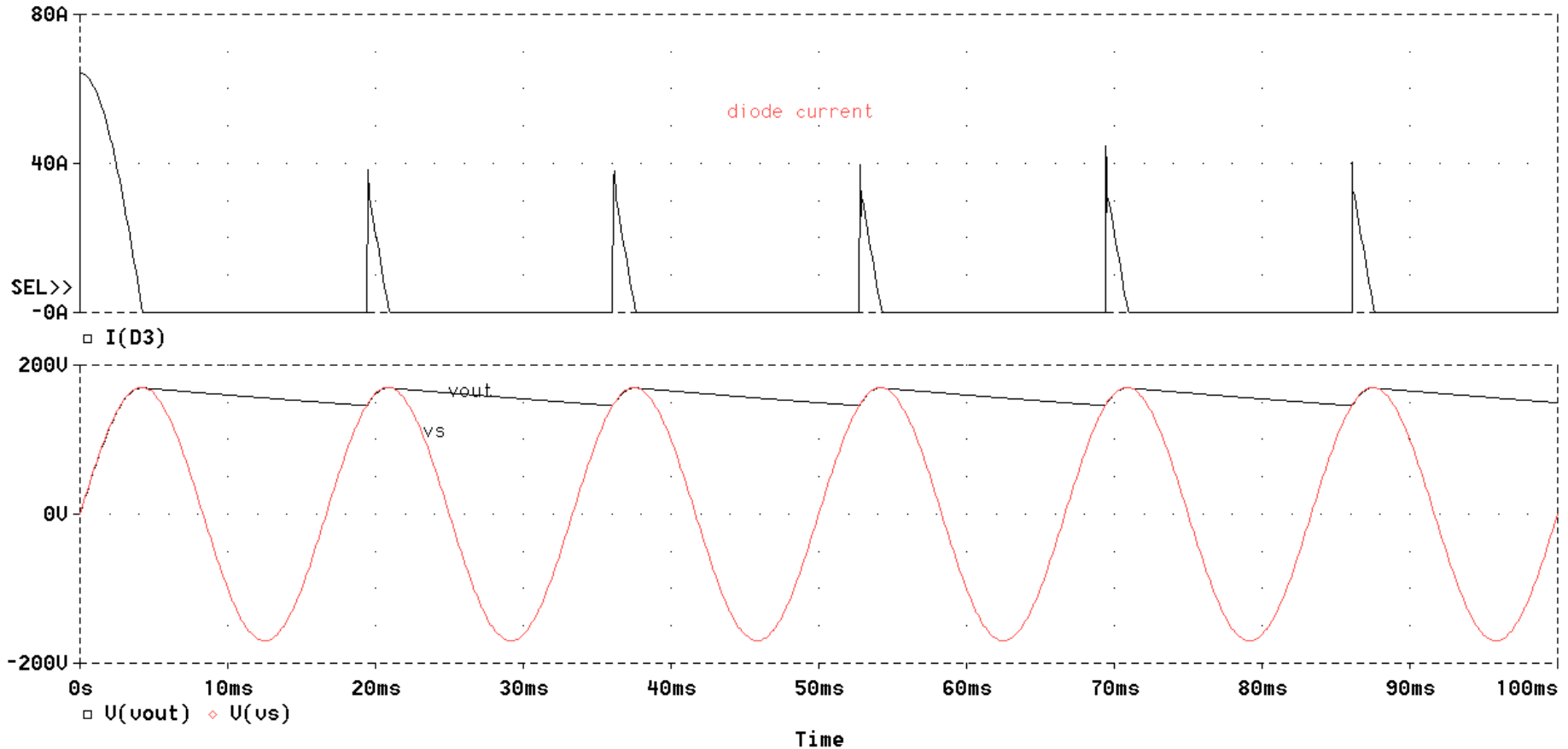
Half Wave Rectifier with RC Load

- If $RC \gg 1/f$ then this operates like a peak detector and the output voltage $\langle v_{out} \rangle$ is approximately the peak of the input voltage
- Diode is only ON for a short time near the sinewave peaks



Half Wave Rectifier with RC Load

- Note poor power factor due to peaky input line current



Unity Power Factor --- Resistive Load

- Example: purely resistive load
 - Voltage and currents in phase

$$v(t) = V \sin \omega t$$

$$i(t) = \frac{V}{R} \sin \omega t$$

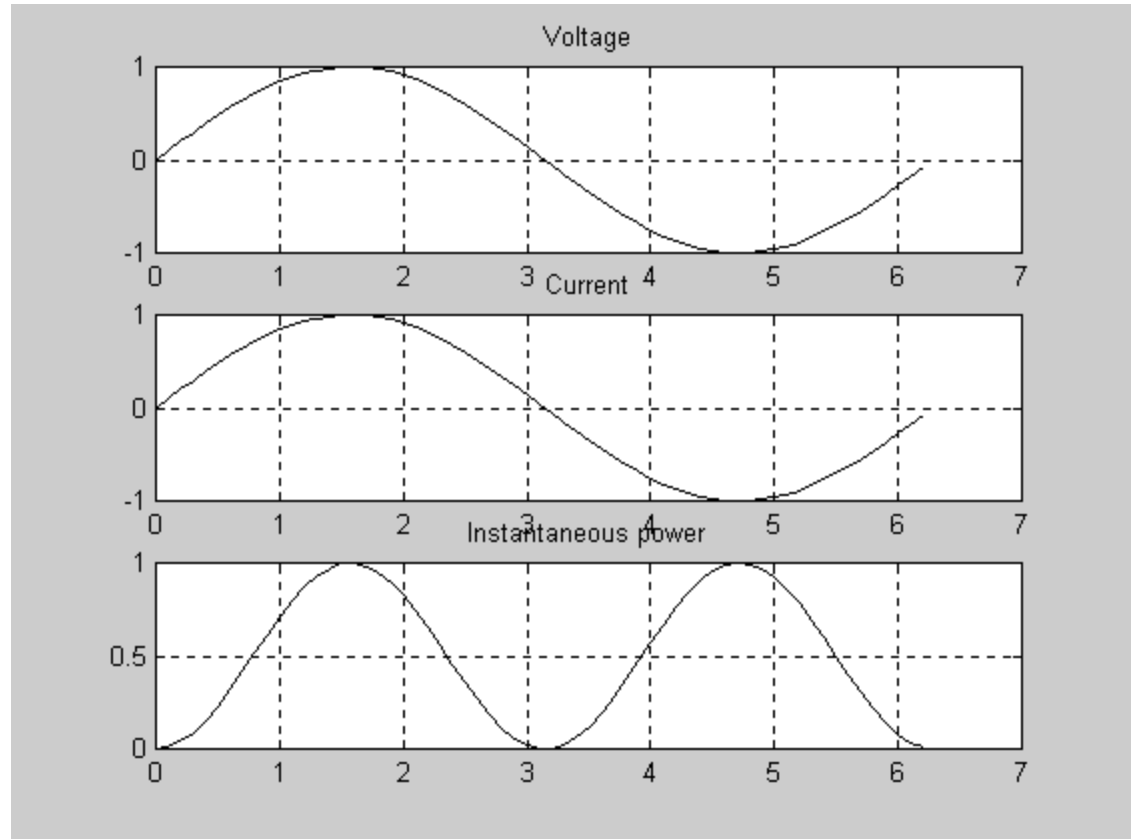
$$p(t) = v(t)i(t) = \frac{V^2}{R} \sin^2 \omega t$$

$$\langle p(t) \rangle = \frac{V^2}{2R}$$

$$V_{RMS} = \frac{V}{\sqrt{2}}$$

$$I_{RMS} = \frac{V}{R\sqrt{2}}$$

$$PF = \frac{\langle p(t) \rangle}{V_{RMS} I_{RMS}} = \frac{\frac{V^2}{2R}}{\left(\frac{V}{\sqrt{2}}\right)\left(\frac{V}{R\sqrt{2}}\right)} = 1$$



Causes of Low Power Factor --- Reactive Load

- Example: purely inductive load
 - Voltage and currents 90° out of phase

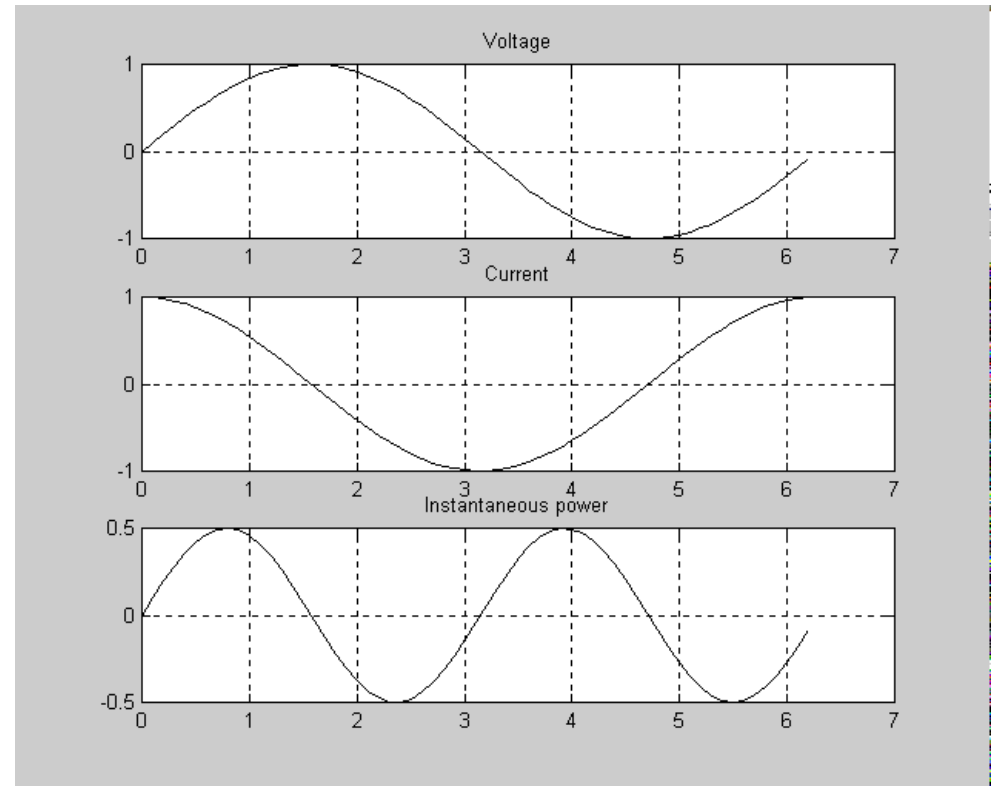
$$v(t) = V \sin \omega t$$

$$i(t) = \frac{V}{\omega L} \cos \omega t$$

$$p(t) = v(t)i(t) = \frac{V^2}{\omega L} \sin \omega t \cos \omega t$$

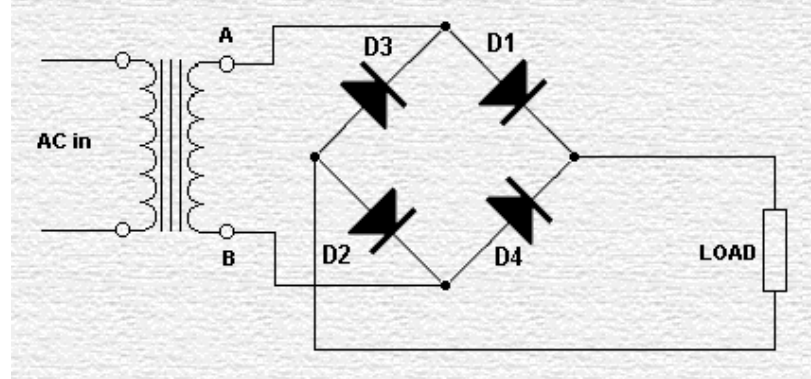
$$\langle p(t) \rangle = 0$$

- For purely reactive load,
PF=0



Why is Power Factor Important?

- Consider peak-detector full-wave rectifier



- Typical power factor $k_p = 0.6$
- What is maximum power you can deliver to load ?
 - $V_{AC} \times \text{current} \times k_p \times \text{rectifier efficiency}$
 - $(120)(15)(0.6)(0.98) = 1058 \text{ Watts}$
- Assume you replace this simple rectifier by power electronics module with 99% power factor and 93% efficiency:
 - $(120)(15)(0.99)(0.93) = 1657 \text{ Watts}$

Power Factor Correction

- A toaster can draw 1500W from a 120V/15A line
- Typical offline switching power converter can draw <1000W from the line since it has poor power factor
- High power factor results in: lower utility bills, increased system capacity, better voltage quality, reduced heating losses
- Methods of power factor correction
 - Passive: add capacitors or inductors
 - Active

Power Factor Correction --- Passive

- Switch capacitors in and out as needed as load changes



Power Factor Corrected Power Supplies

Without PFC

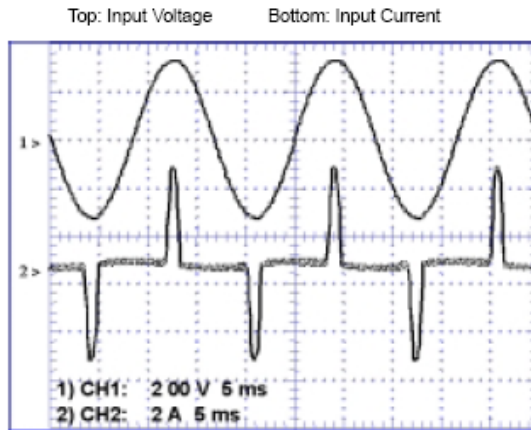


Figure 1. Input Characteristics of a Typical Switched-Mode Power Supply without PFC

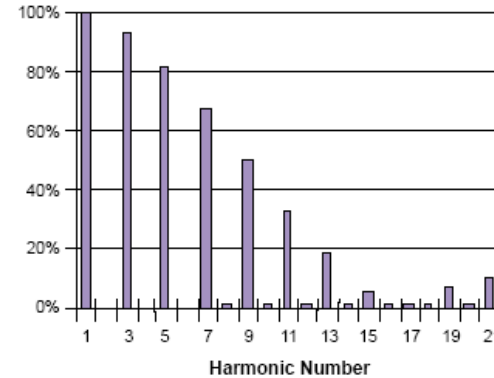


Figure 2. Harmonic Content of the Current Waveform in Figure 1

With PFC

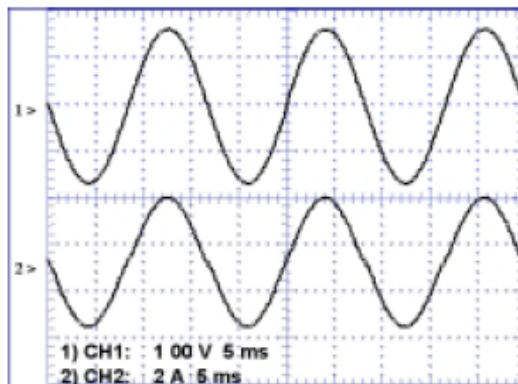
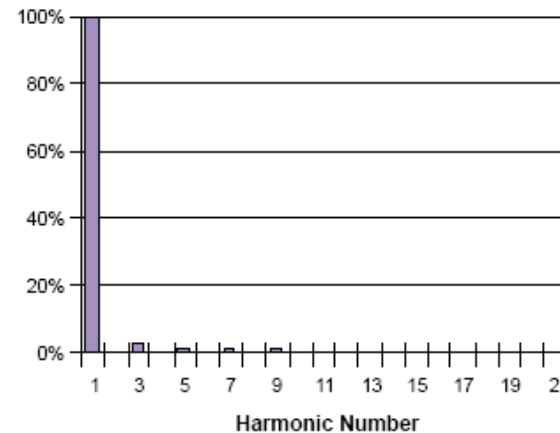


Figure 3. Input Characteristics of a Power Supply with Near-Perfect PFC



Reference: On Semiconductor, "Power Factor Correction Handbook," www.onsemi.com

Power Supply with Power Factor Correction

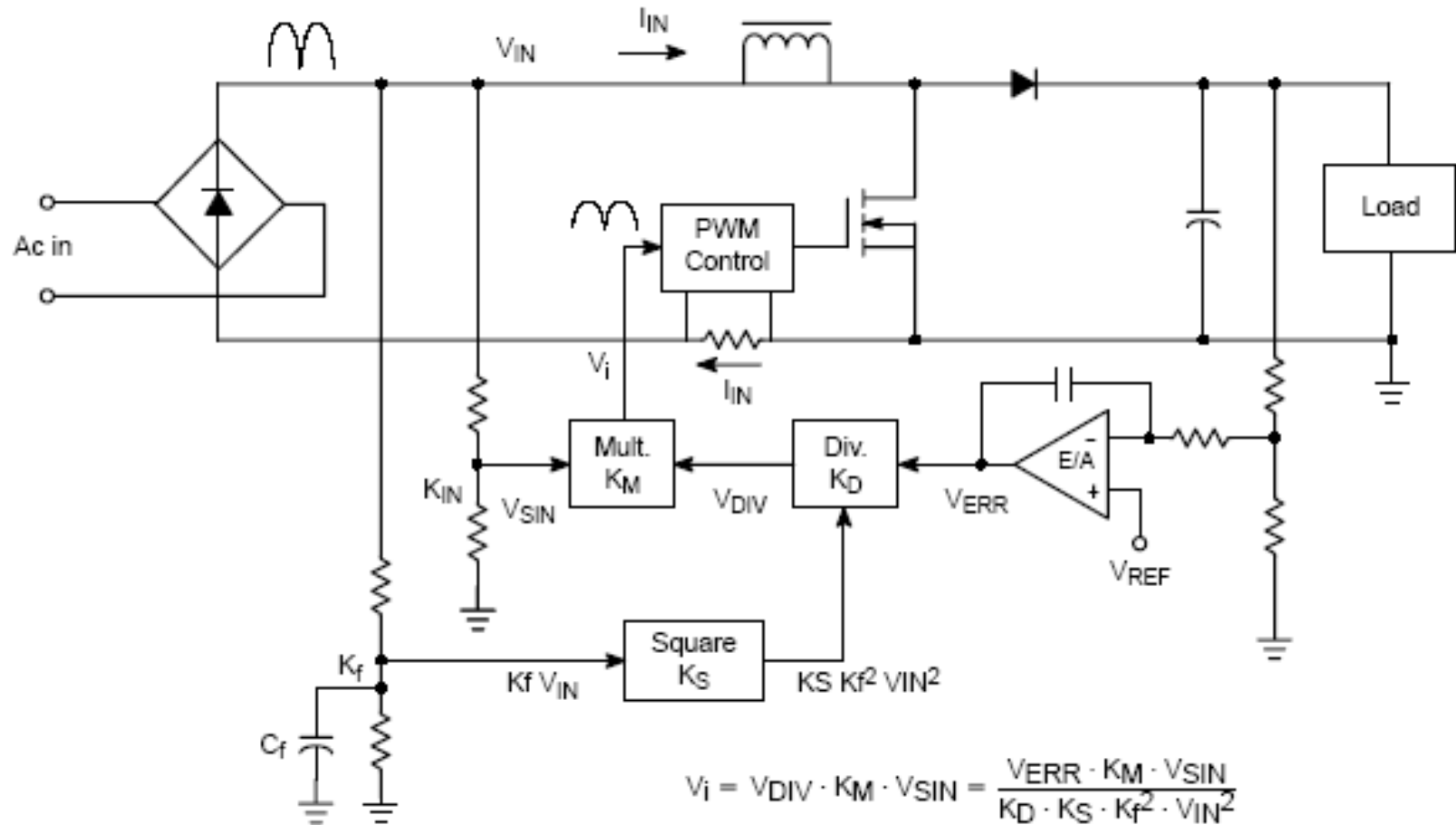


Figure 12. Block Diagram of the Classic PFC Circuit

Reference: On Semiconductor, "Power Factor Correction Handbook," www.onsemi.com

Harmonics

- Joseph Fourier (1768-1830) worked out that any periodic waveform can be broken up into a fundamental frequency plus harmonics
- The harmonics need to have the proper amplitude and phase relationship to the fundamental
- The result is a “Fourier series” for a periodic waveform

Fourier Series

- Here's the recipe:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

- Finding the Fourier coefficients:

$$a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx$$

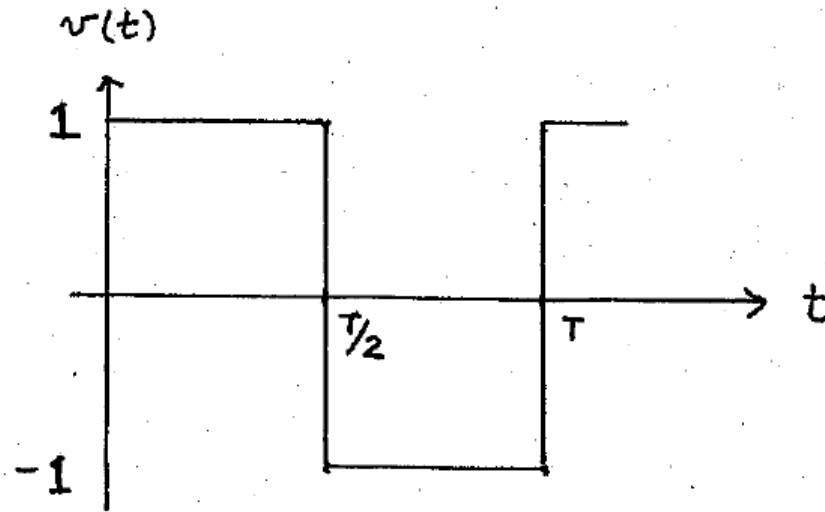
$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx \quad n = 1, 2, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx \quad n = 1, 2, \dots$$

Reference: http://www.efunda.com/math/fourier_series/fourier_series.cfm

Harmonics of Square Wave

- A 50% duty cycle square wave:

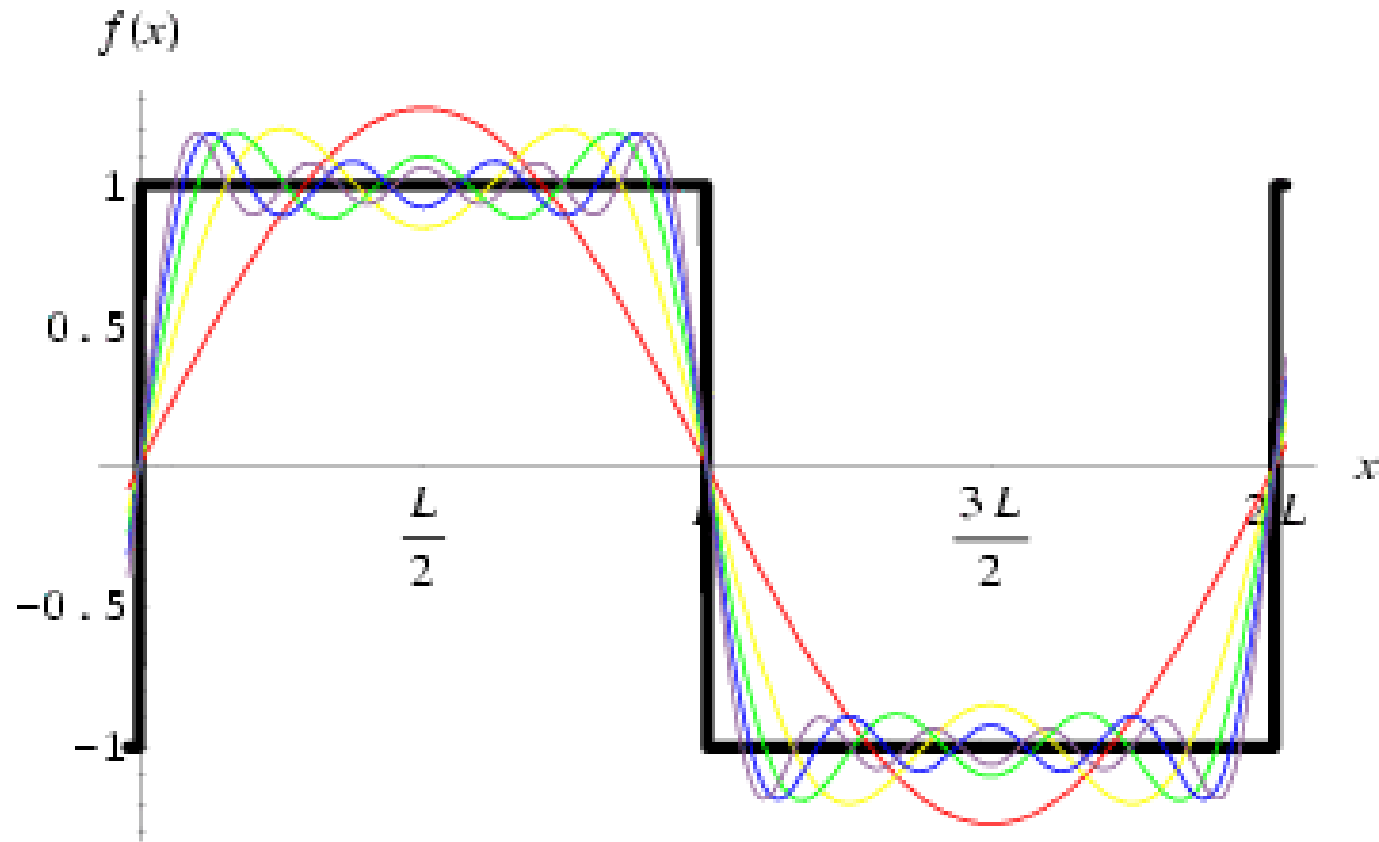


$$v(t) = \left(\frac{4}{\pi}\right)\sin(\omega t) + \left(\frac{4}{3\pi}\right)\sin(3\omega t) + \left(\frac{4}{5\pi}\right)\sin(5\omega t) + \dots$$

$$\omega = \frac{2\pi}{T}$$

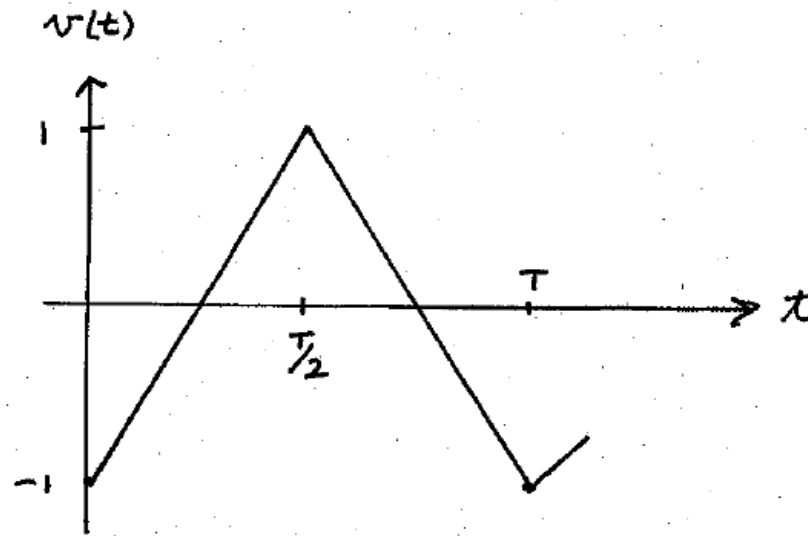
Building Up a Square Wave

- Building up a 50% duty cycle square wave:



Reference: <http://mathworld.wolfram.com/FourierSeries.html>

Triangle Wave



$$v(t) = \left(\frac{8}{\pi^2}\right)\sin(\omega t) - \left(\frac{8}{3^2 \pi^2}\right)\sin(3\omega t) + \left(\frac{8}{5^2 \pi^2}\right)\sin(5\omega t) + \dots$$

Harmonics in the Power Line

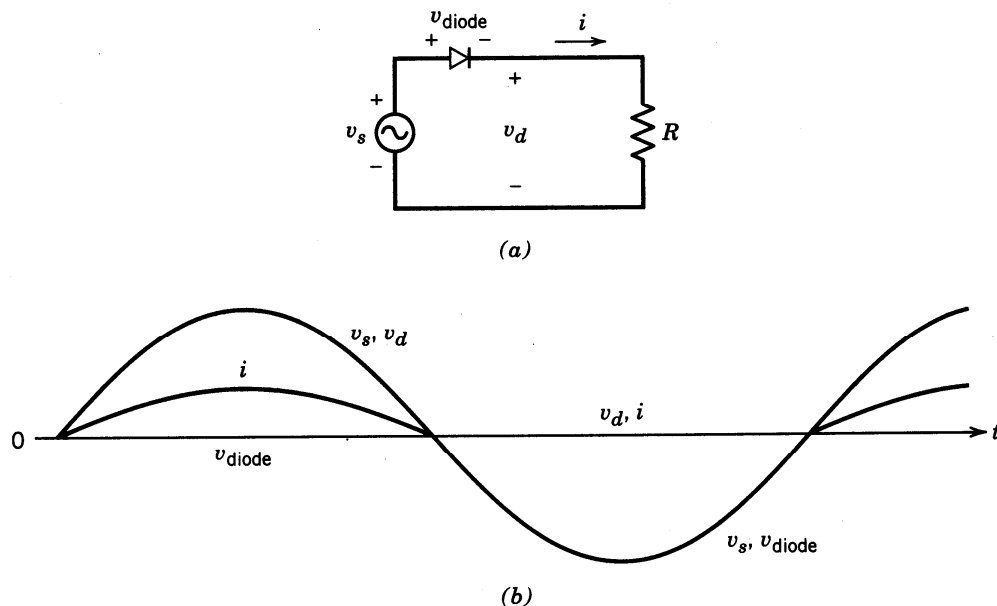
- Harmonics are created by nonlinear circuits
 - Rectifiers
 - Half-wave rectifier has first harmonic at 60 Hz
 - Full-wave has first harmonic at 120 Hz
 - Switching DC/DC converters
 - DC/DC operating at 100 kHz generally creates harmonics at DC, 100 kHz, 200 kHz, 300 kHz, etc.
- Line harmonics can be treated by line filters
 - Passive
 - Active

Harmonics in the Power Line

- The issue of harmonics has become much more important in recent years in power systems
- Harmonic sources include
 - Switching power supplies
 - Variable speed drives (VSDs)
 - Arc furnaces
 - Welders
 - Fluorescent lamp ballasts

Half-Wave Rectifier, Resistive Load

- Simplest, cheapest rectifier
- Line current has DC component; this current appears in neutral
- High harmonic content, Power factor = 0.7



$$P.F. = \frac{P_{avg}}{V_{RMS} I_{RMS}}$$

Figure 5-2 Basic rectifier with a load resistance.

Reference: Mohan, Undeland and Robbins, *Power Electronics, Converters, Applications and Design*, John Wiley, 2003, pp. 80

Half Wave Rectifier with Resistive Load --- Power Factor and Average Output Voltage

Average output voltage:

$$\langle v_d \rangle = \frac{1}{2\pi} \int_0^\pi V_{pk} \sin(\omega t) d(\omega t) = \frac{V_{pk}}{2\pi} [-\cos(\omega t)]_{\omega t=0}^{\omega t=\pi} = \frac{V_{pk}}{\pi}$$

Power factor calculation:

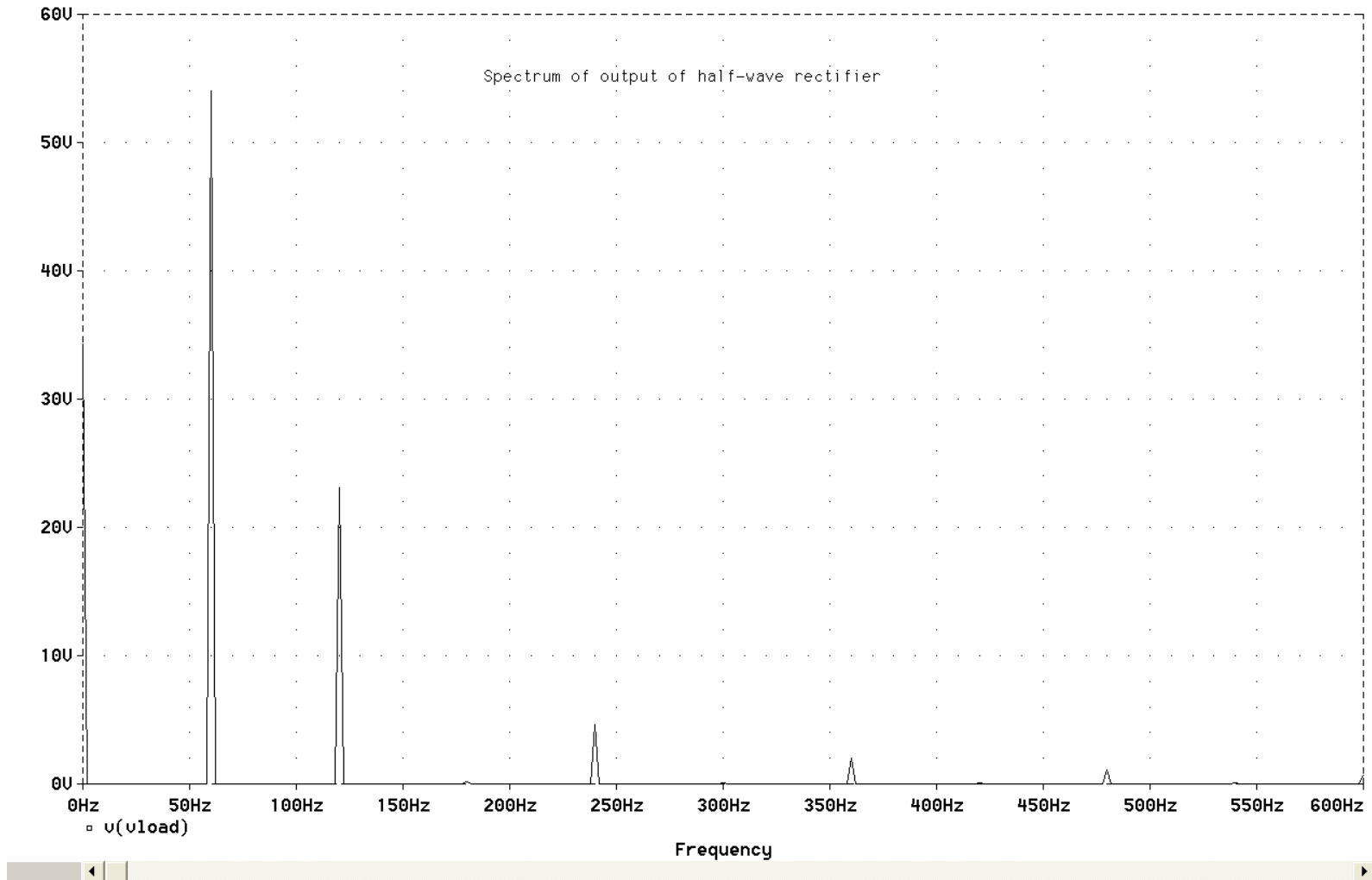
$$\langle P \rangle = \frac{1}{2} \left(\frac{I_{pk}}{\sqrt{2}} \right)^2 R = \frac{I_{pk}^2}{4} R$$

$$V_{RMS} = \frac{V_{pk}}{\sqrt{2}} = \frac{I_{pk}}{R\sqrt{2}}$$

$$I_{RMS} = \frac{I_{pk}}{\sqrt{2}} \frac{1}{\sqrt{2}}$$

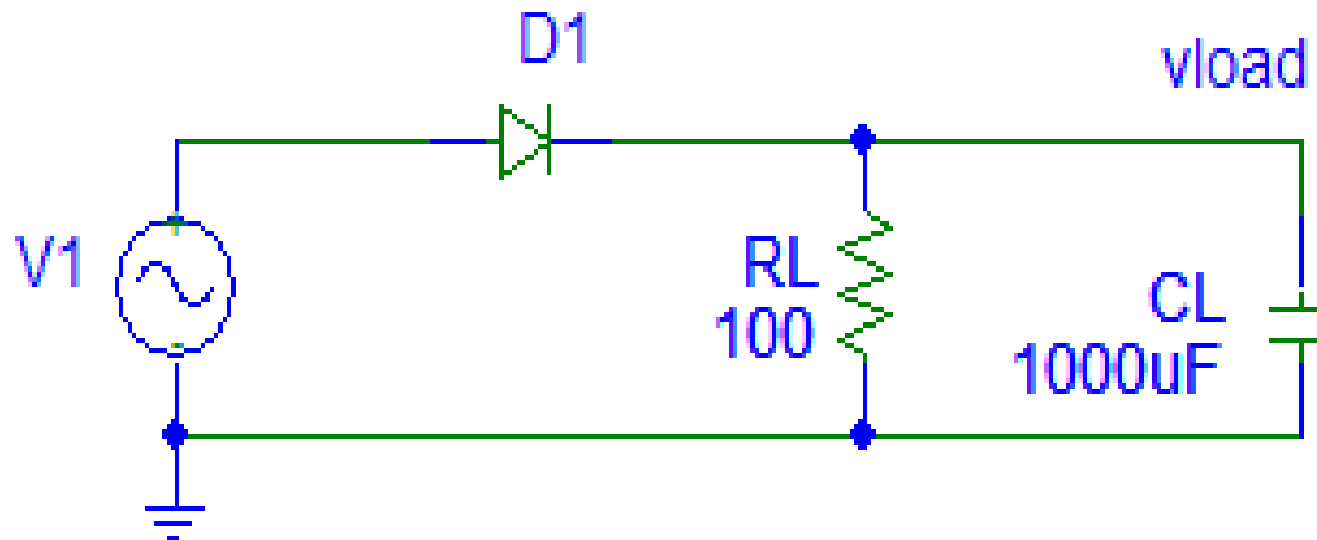
$$PF = \frac{\langle P \rangle}{V_{RMS} I_{RMS}} = \frac{\frac{I_{pk}^2}{4} R}{\left(\frac{I_{pk}}{R\sqrt{2}} \right) \left(\frac{I_{pk}}{\sqrt{2}} \frac{1}{\sqrt{2}} \right)} = 0.707$$

Half-Wave Rectifier, Resistive Load --- Spectrum of Load Voltage



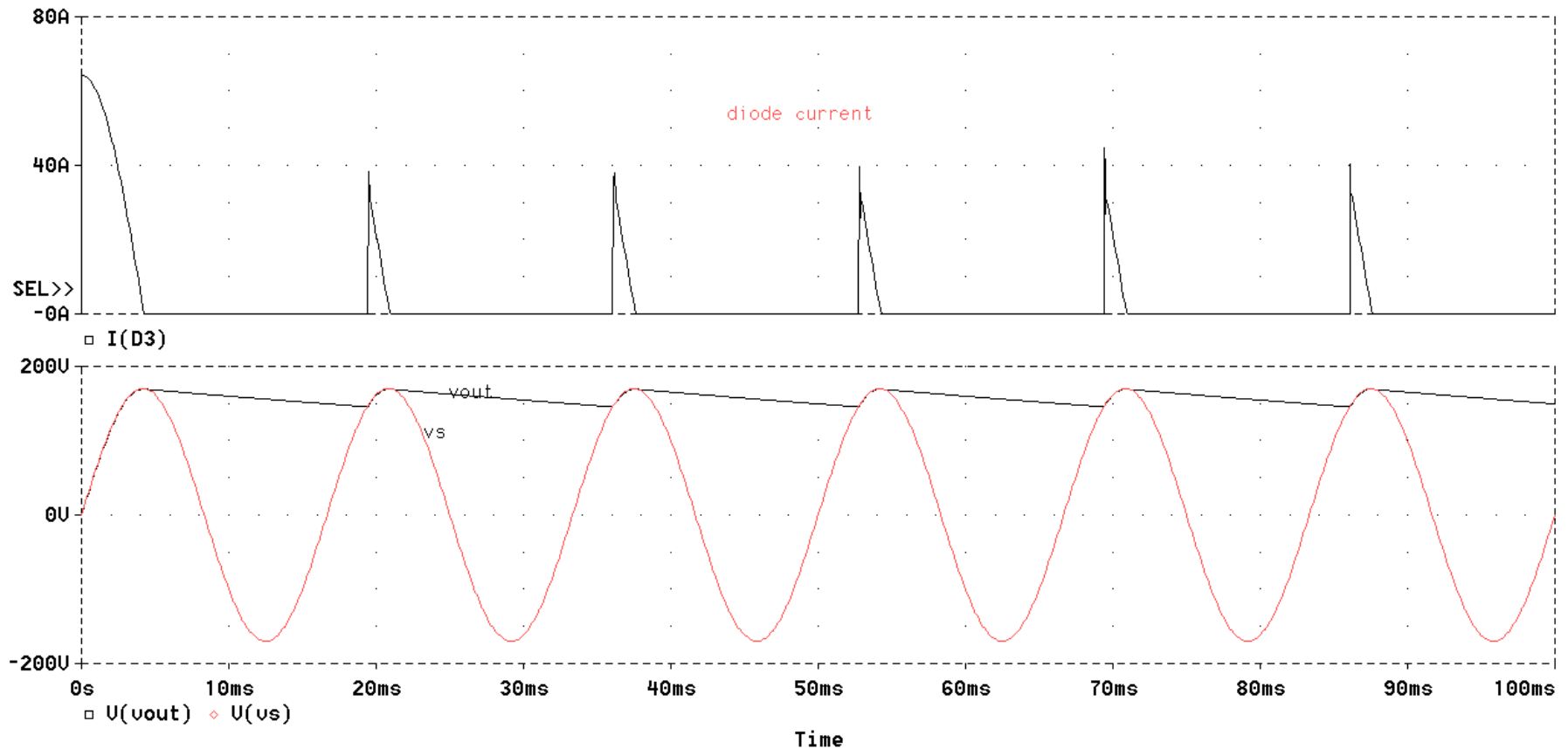
Half Wave Rectifier with RC Load

- More practical rectifier
- For large RC, this behaves like a peak detector

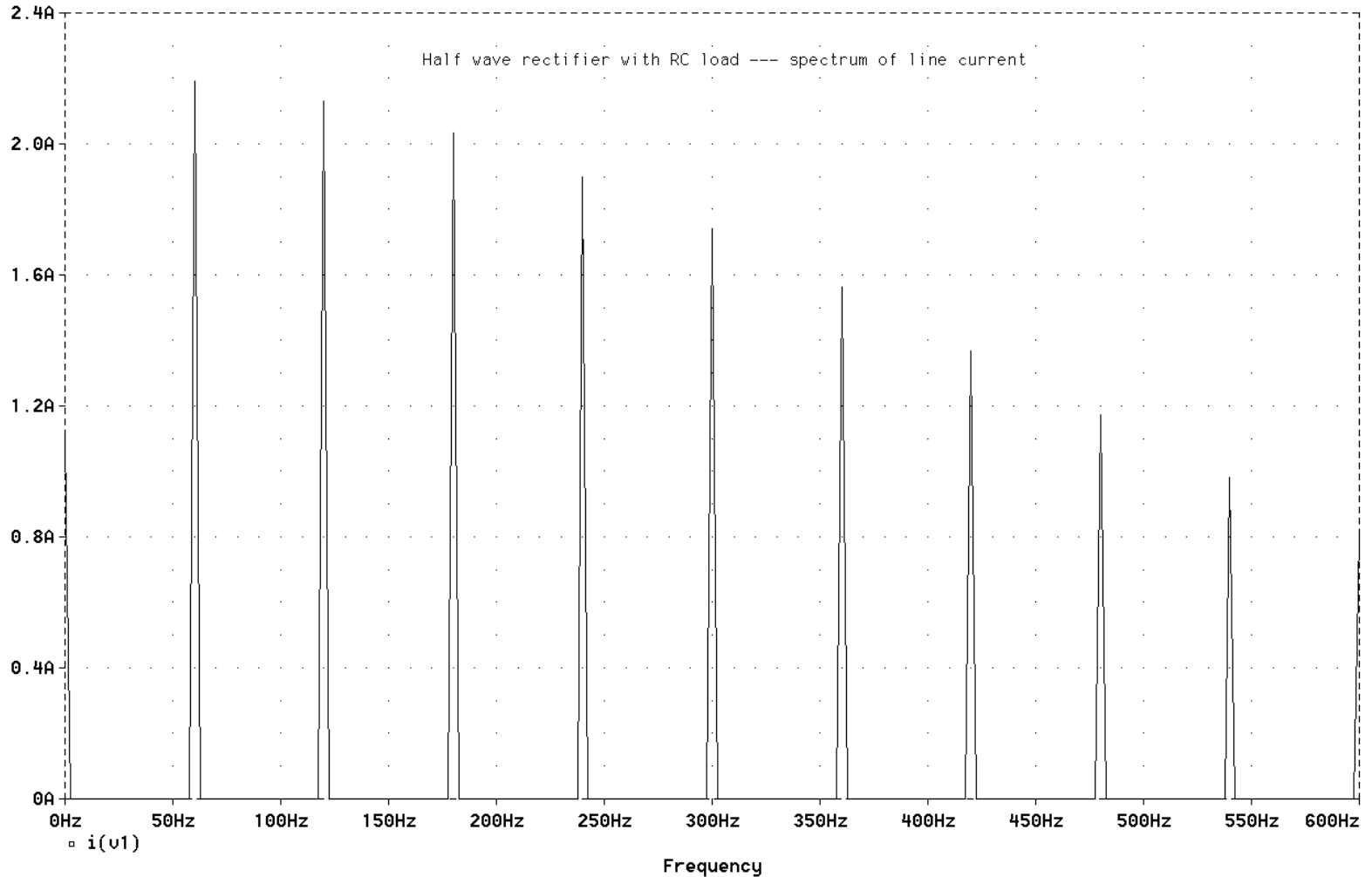


Half Wave Rectifier with RC Load

- Note poor power factor due to peaky line current
- Note DC component of line current



Half Wave Rectifier with RC Load --- Spectrum of Line Current



Total Harmonic Distortion

- Total harmonic distortion (THD)
 - Ratio of the RMS value of all the nonfundamental frequency terms to the RMS value of the fundamental

$$THD = \sqrt{\frac{\sum_{n \neq 1} I_{n,RMS}^2}{(I_{1,RMS})^2}} = \sqrt{\frac{I_{RMS}^2 - I_{1,RMS}^2}{I_{1,RMS}^2}}$$

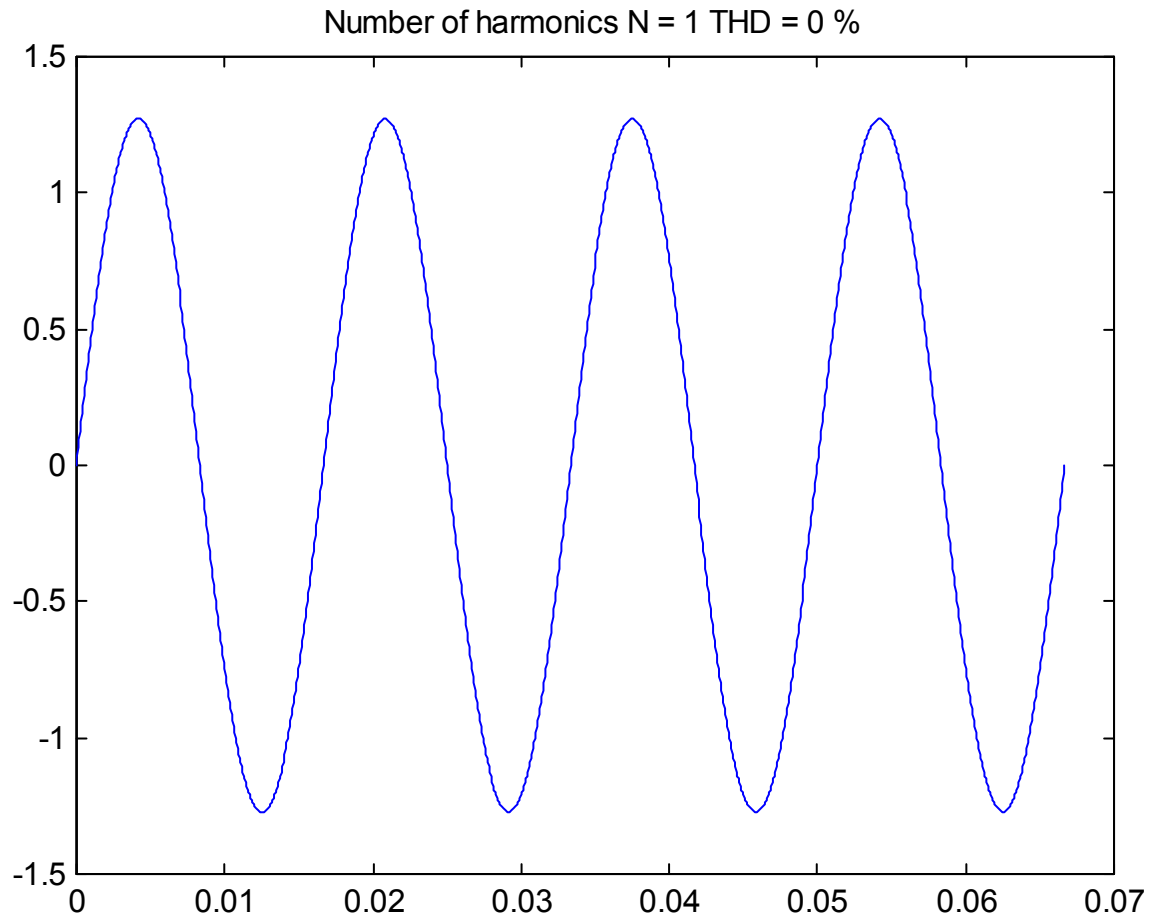
- Symmetrical square wave: THD = 48.3%
- Symmetrical triangle wave: THD=12.1%

Crest Factor

- Another term sometimes used in power engineering
- Ratio of peak value to RMS value
- For a sinewave, crest factor = 1.4
 - Peak = 1; RMS = 0.707
- For a square wave, crest factor = 1
 - Peak = 1; RMS = 1

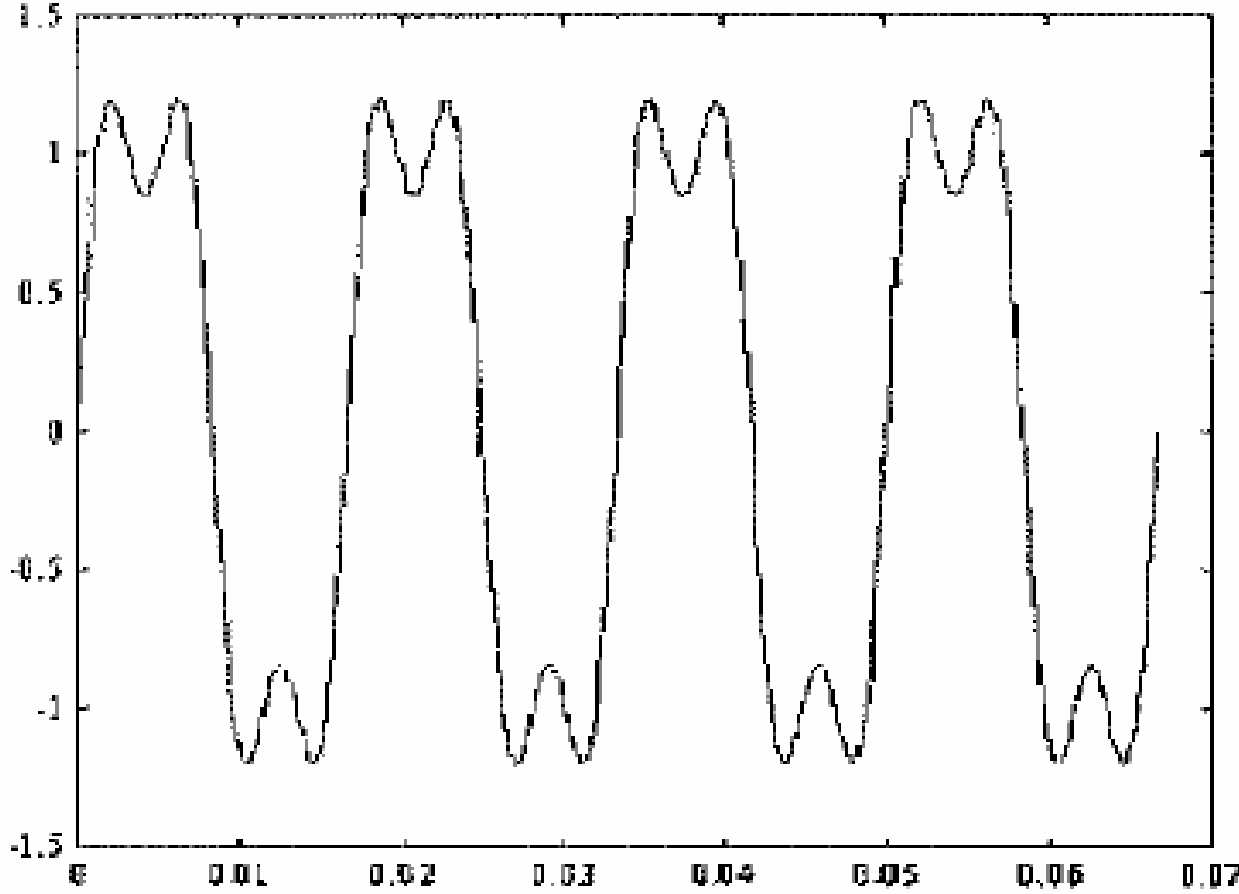
Harmonics and THD --- Pure Sinewave

- THD = 0%



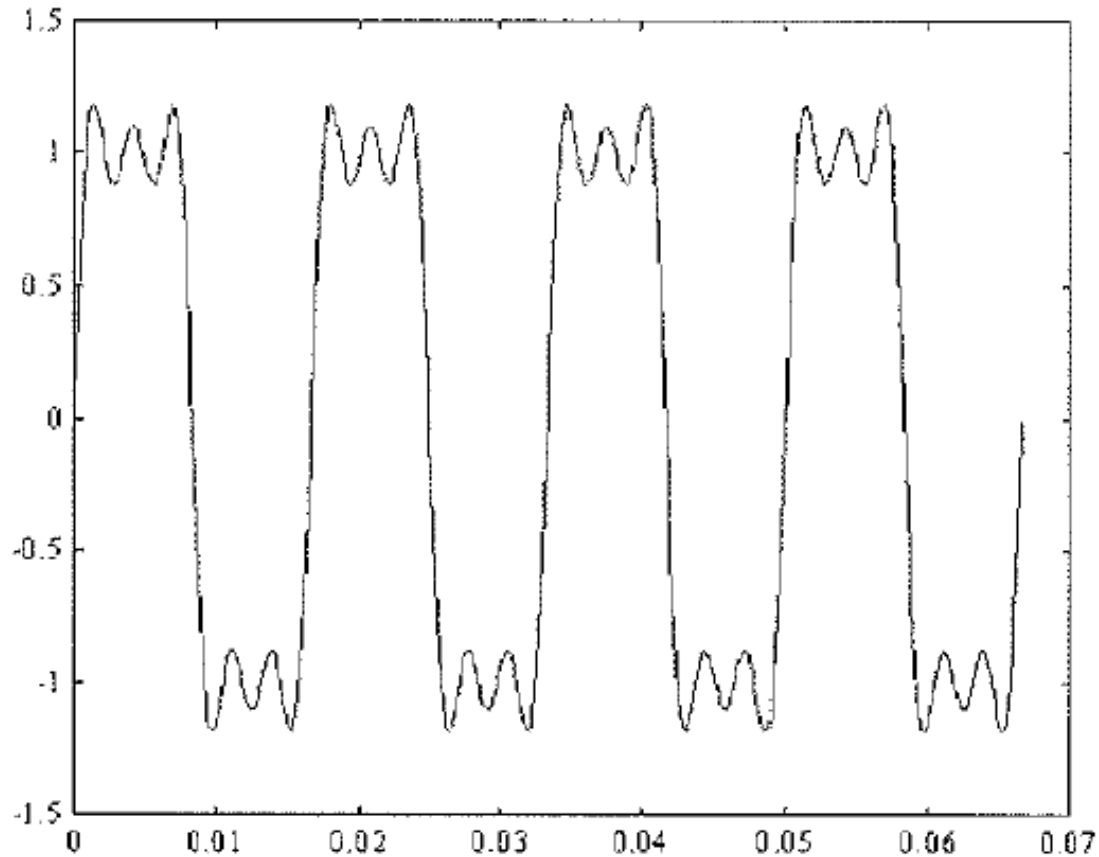
Harmonics and THD - Sinewave + 3rd Harmonic

- THD = 33.3%



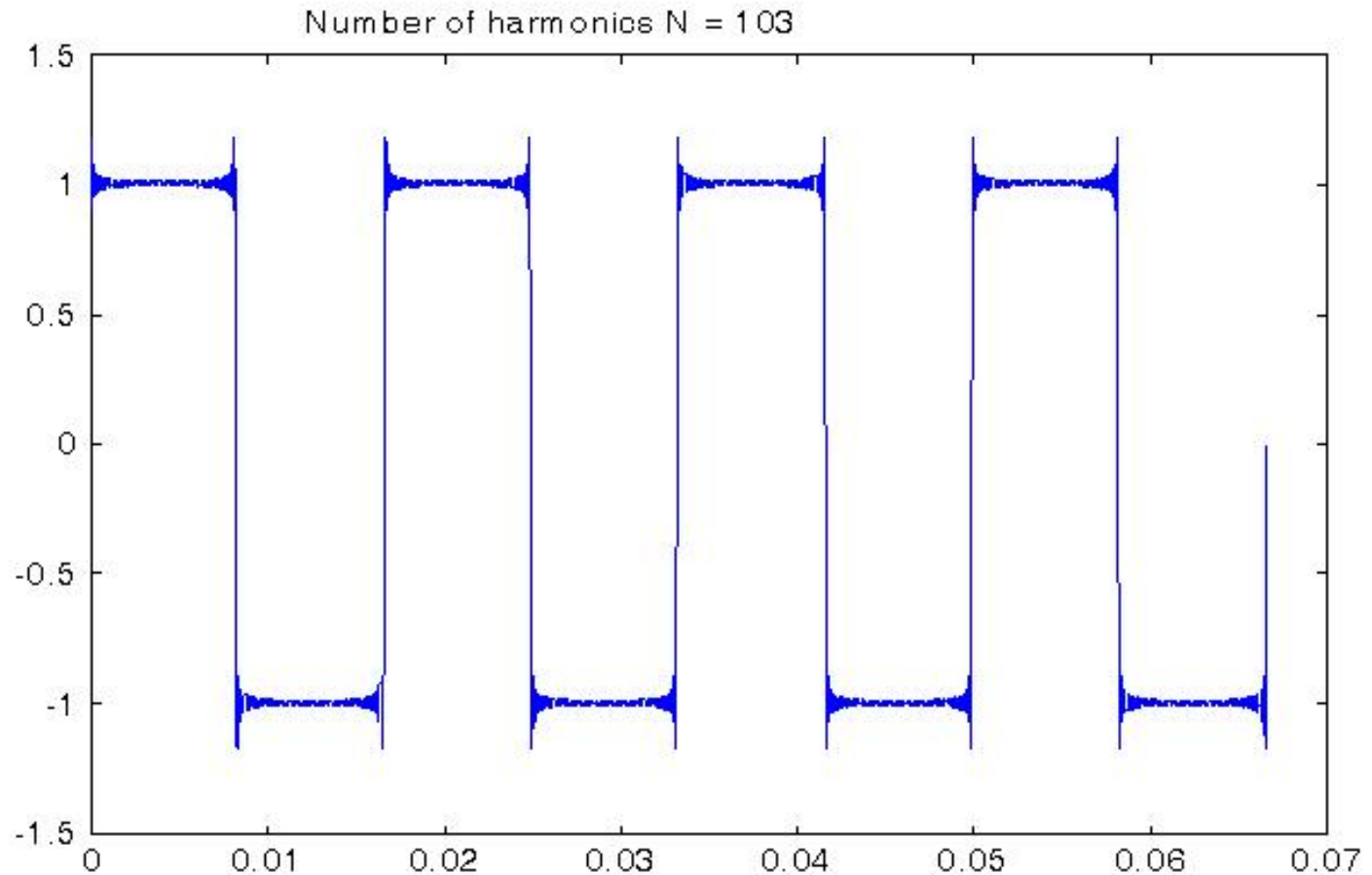
Harmonics and THD --- Sinewave + 3rd + 5th Harmonic

- THD = 38.9%



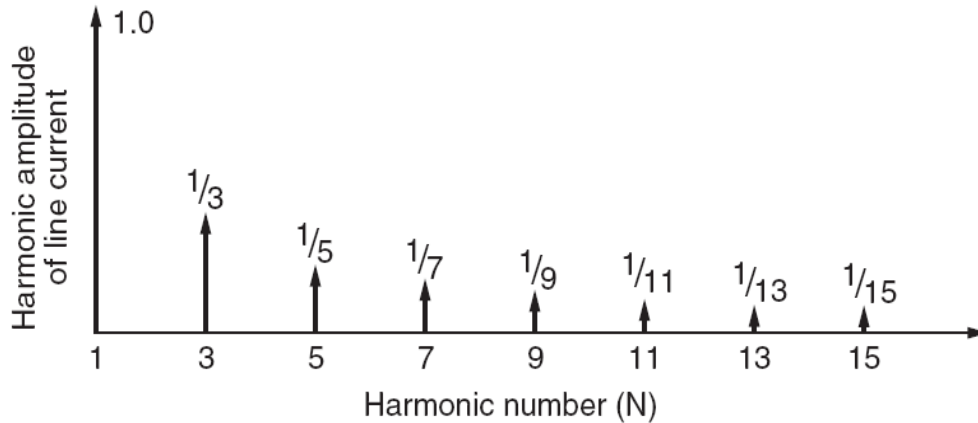
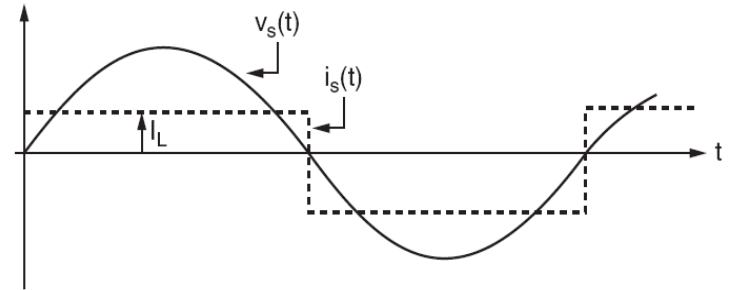
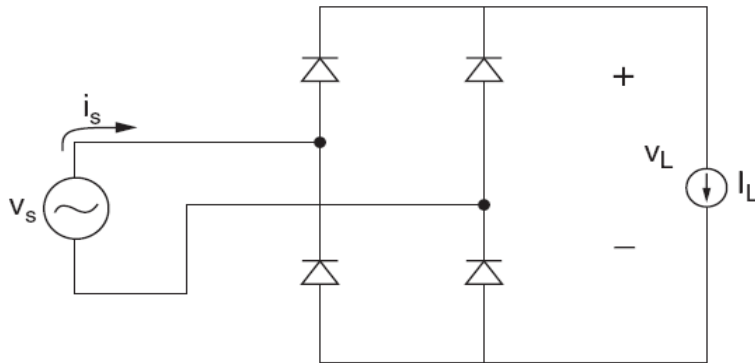
Harmonics --- Up to N = 103

- THD = 48%

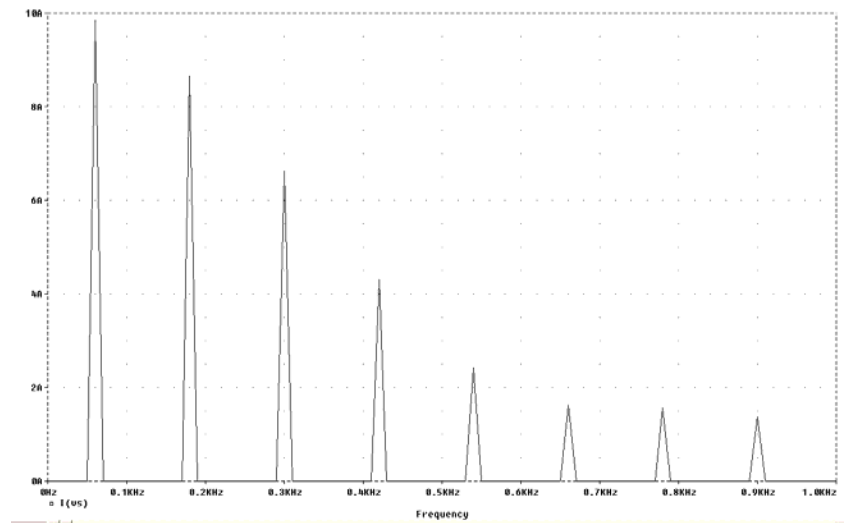
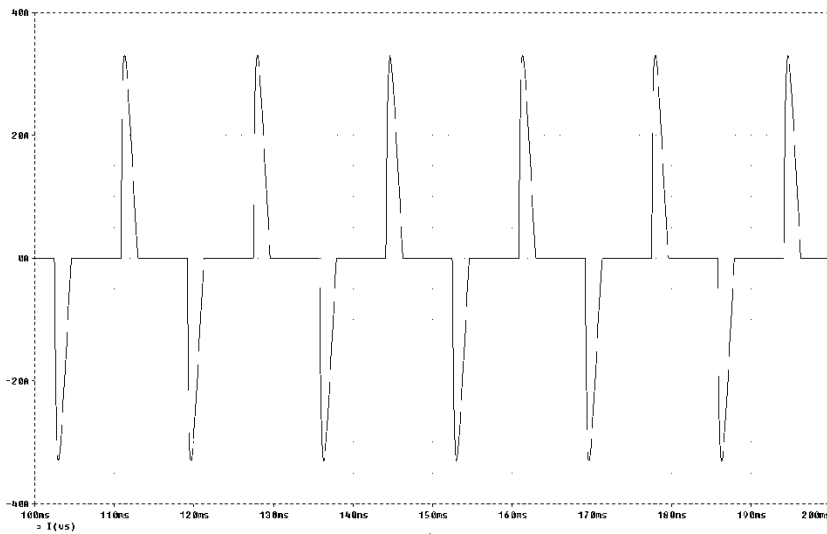
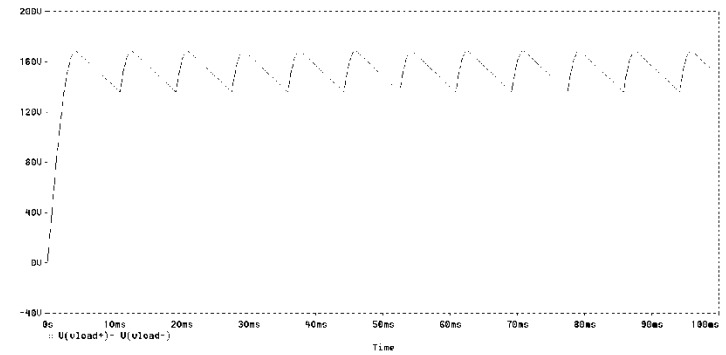
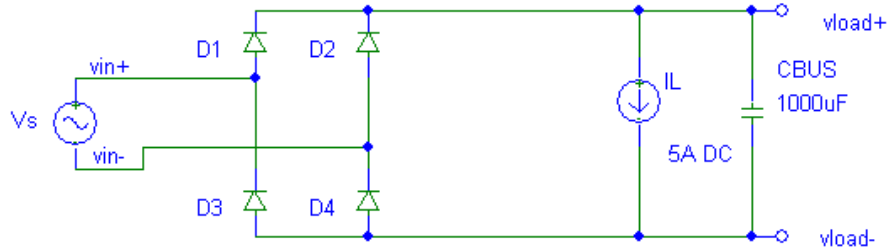


Single Phase, Full-Wave Rectifier

- Draws significant harmonics from the line



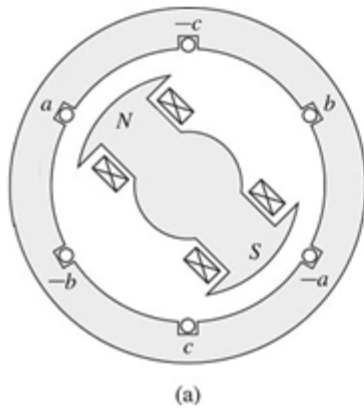
Full-Wave Rectifier with RC Filter --- PSPICE



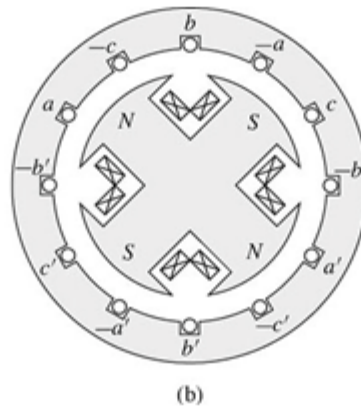
Three-Phase Circuits and Three-Phase Power

- Three phase is used extensively in high power applications
- For high power, it has multiple advantages over single phase

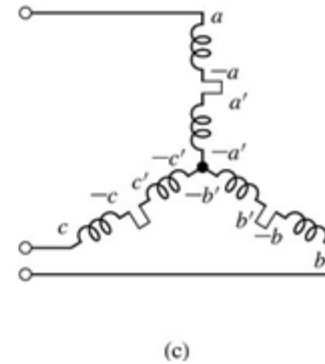
2-pole



4-pole



Y-connection
of windings



A Three-Phase, Four-Wire System

- A common neutral wire is assumed
- If there are 3 equal linear loads, $i_n = 0$
- However, if there are nonlinear loads, there can be a neutral current

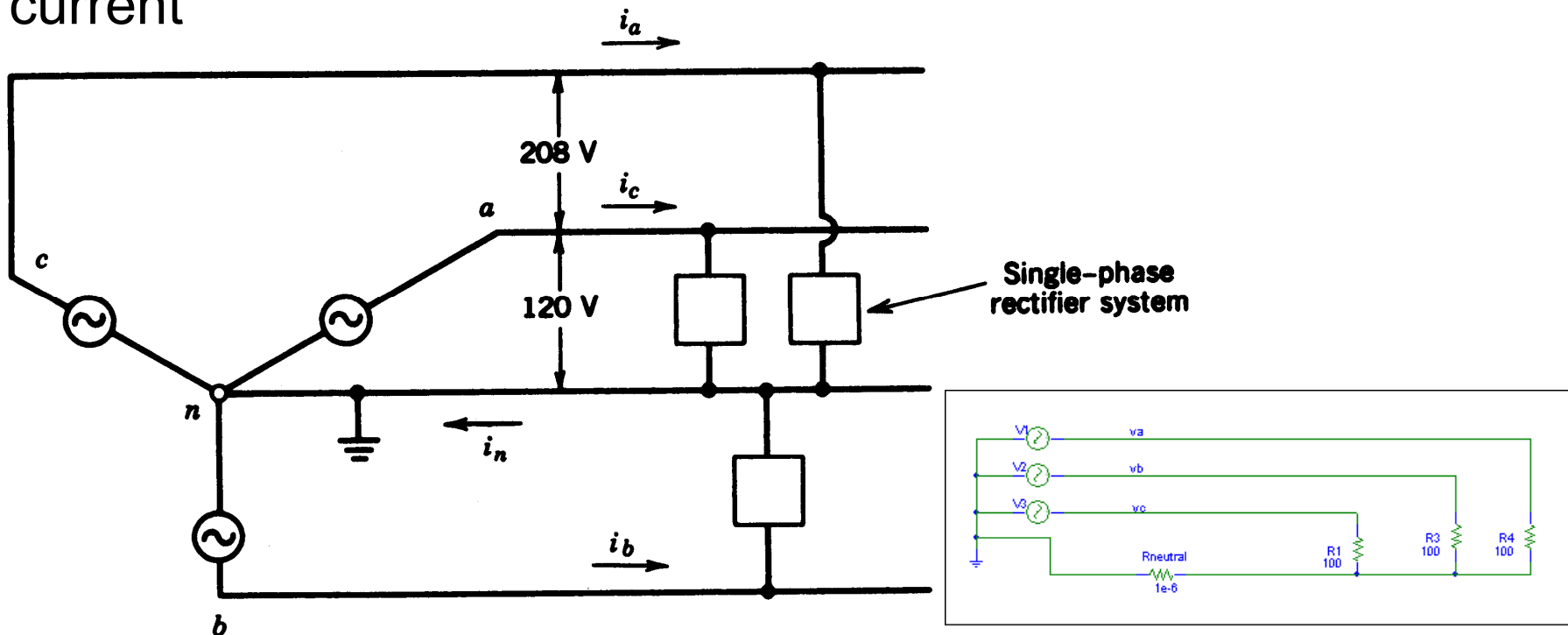
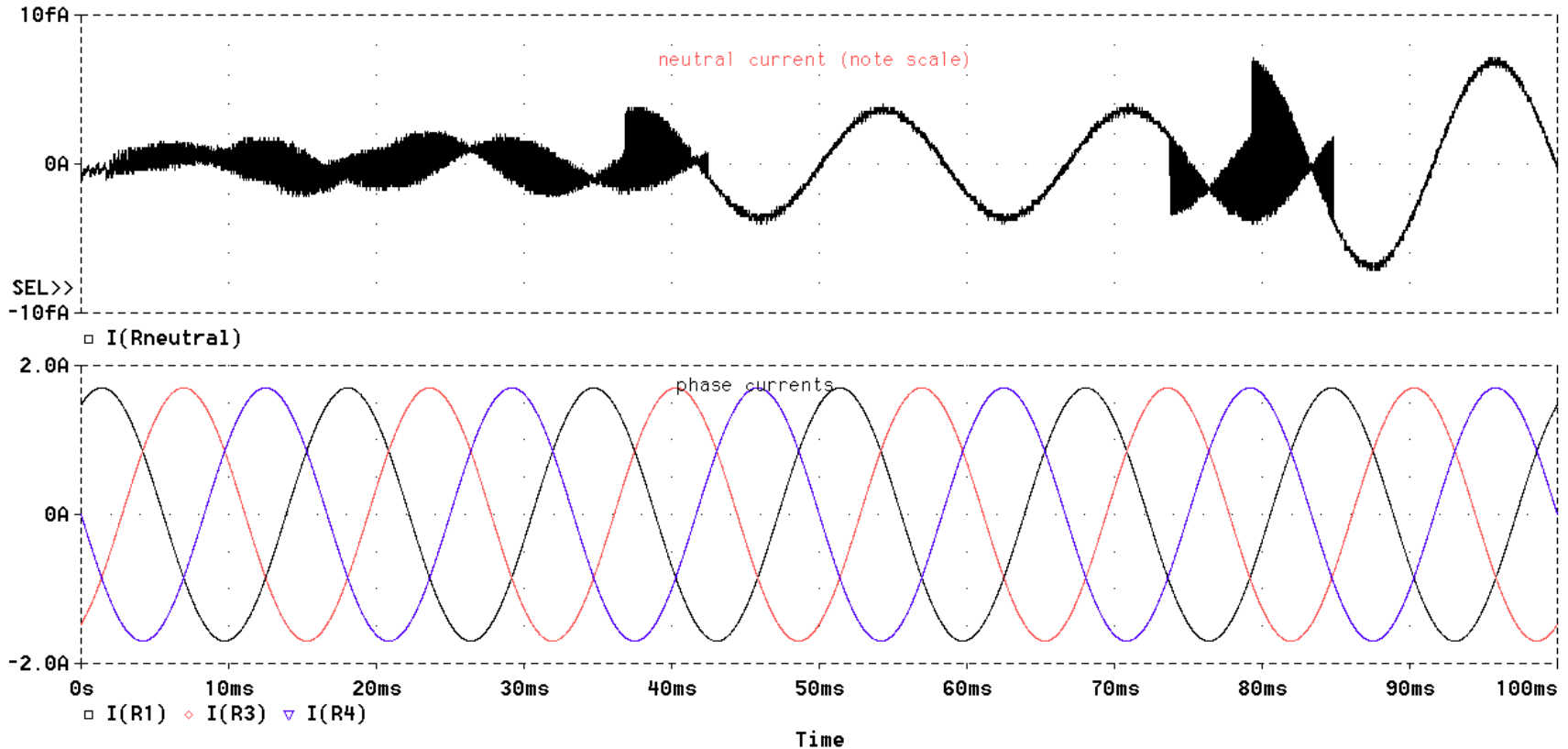


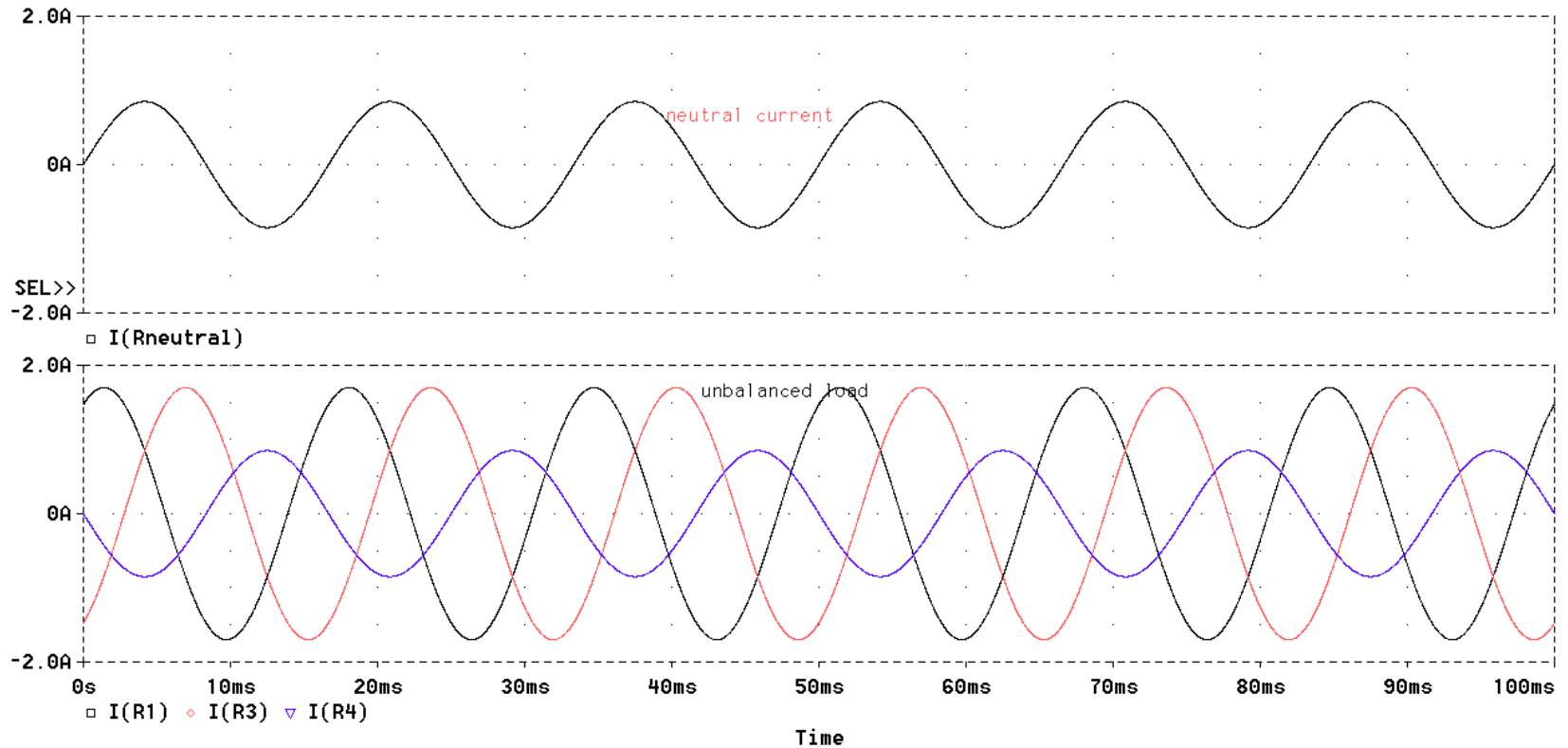
Figure 5-28 Three-phase, four-wire system.

Balanced, Three-Phase, Four-Wire System

- Result shows that for balanced load, $i_n = 0$



Three-Phase, Four-Wire System, Unbalanced Load



Three-Phase, Full-Bridge Rectifier

- Commonly used in high power applications
- Also called “6-pulse rectifier”
- This circuit generates line harmonics

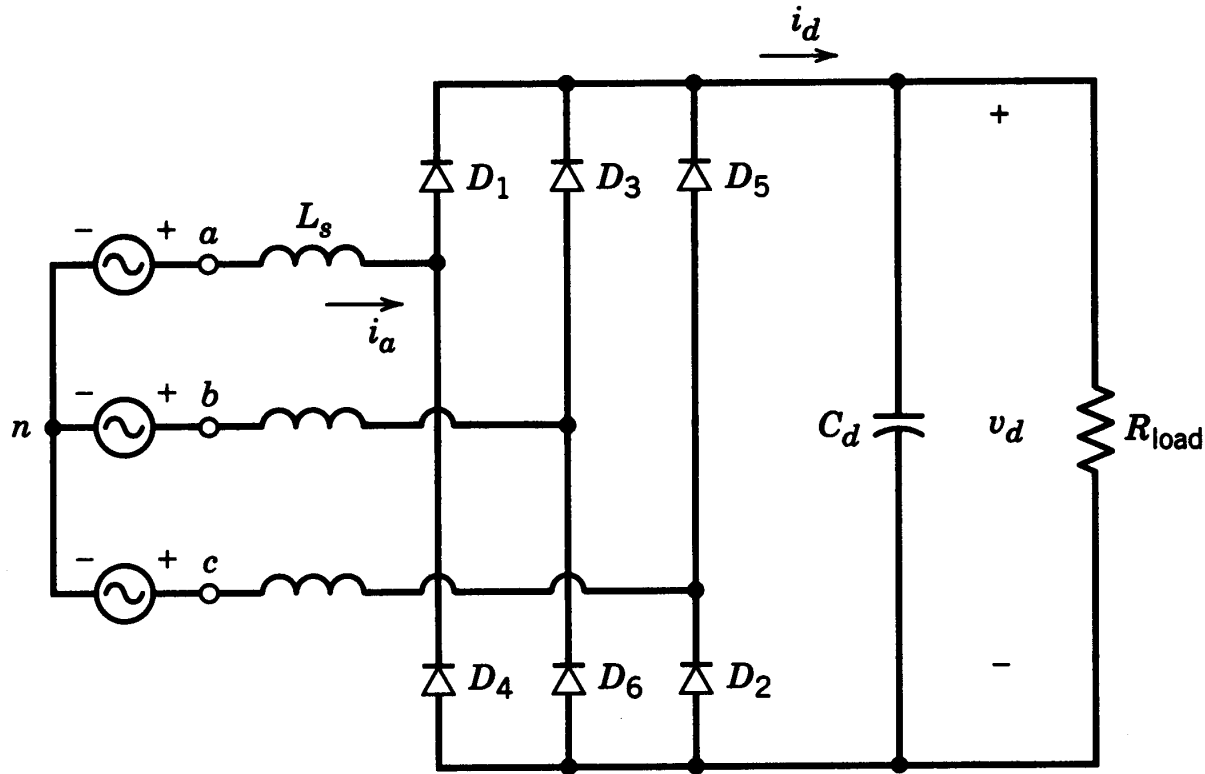
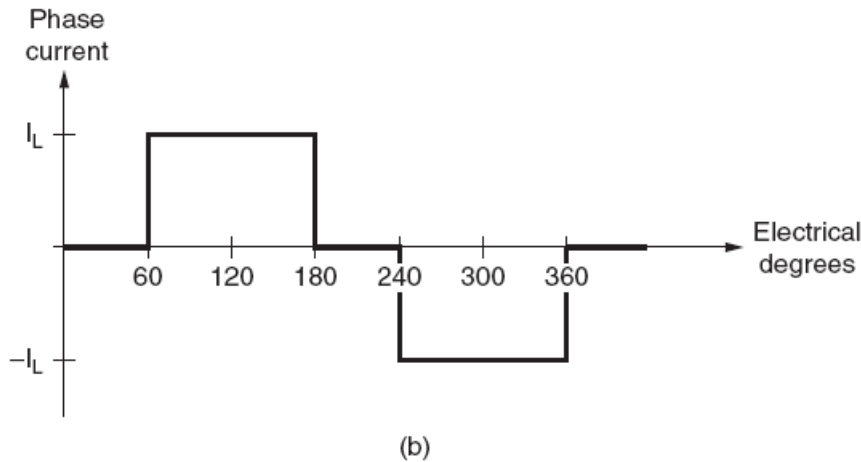
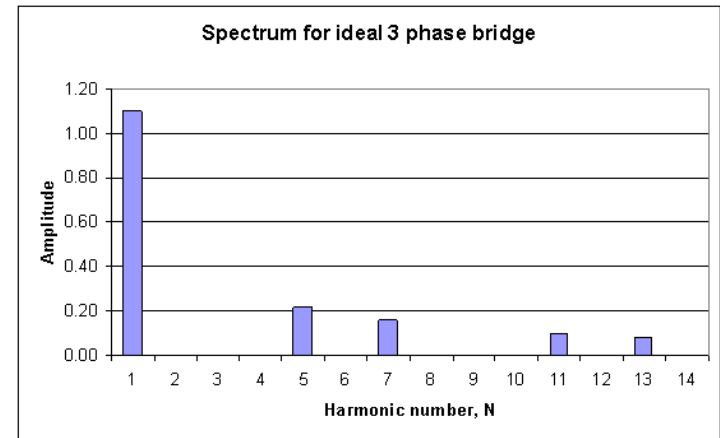
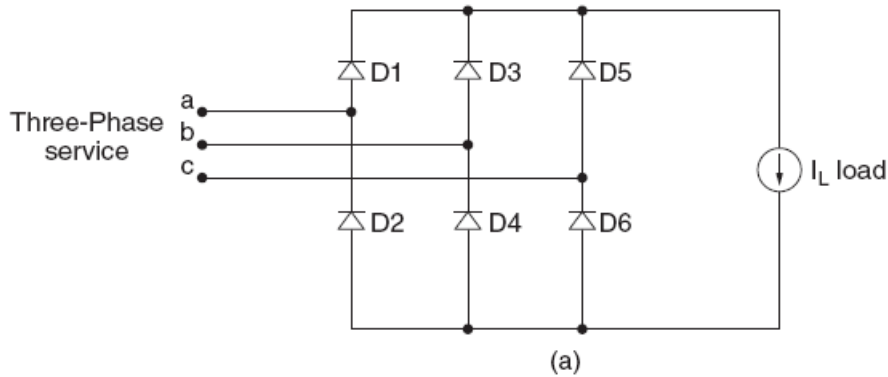


Figure 5-30 Three-phase, full-bridge rectifier.

6-Pulse Rectifier with Current Source Load

- 3-phase source; draws 5th, 7th, 11th, 13th, ... harmonics



6-Pulse Rectifier: Redrawn

- Two groups with three diodes each

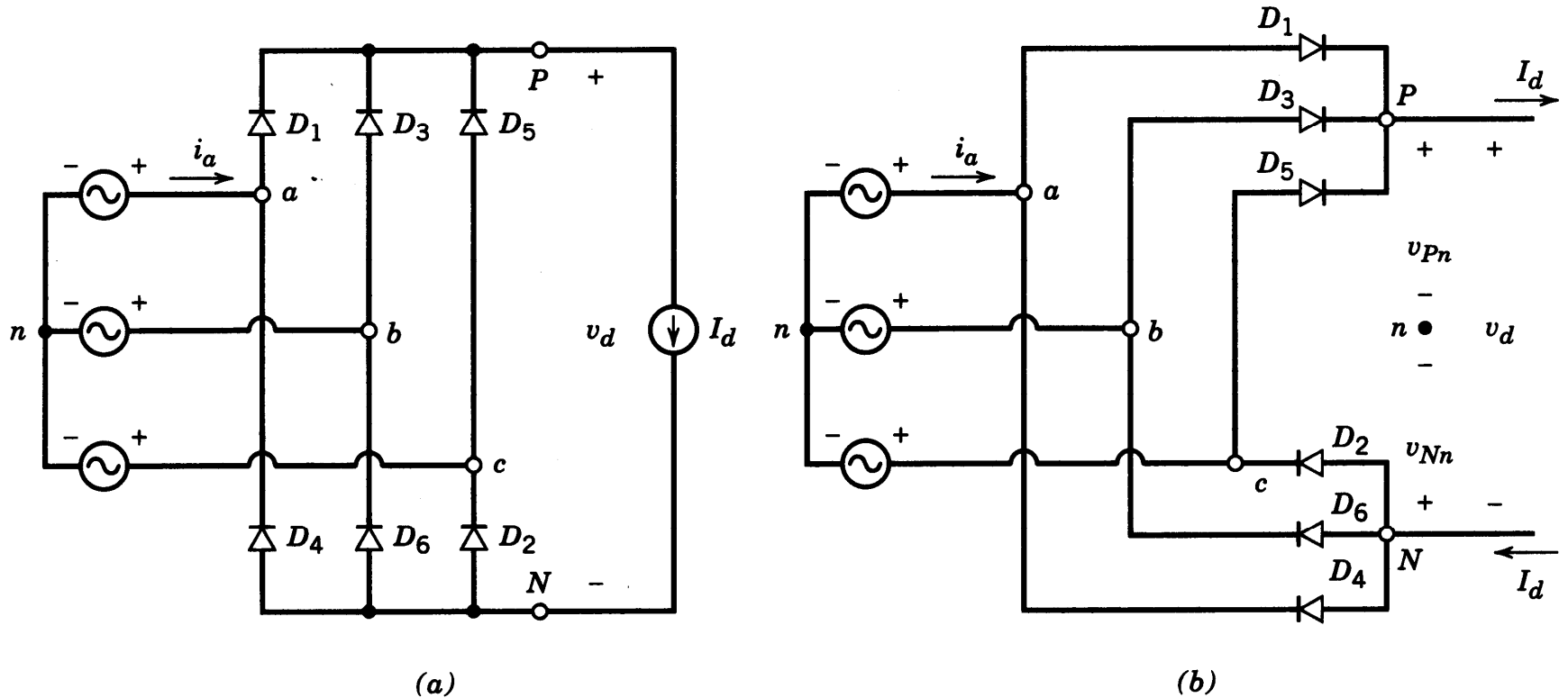


Figure 5-31 Three-phase rectifier with a constant dc current.

6-Pulse Rectifier Waveforms

- Shown for output DC current source load

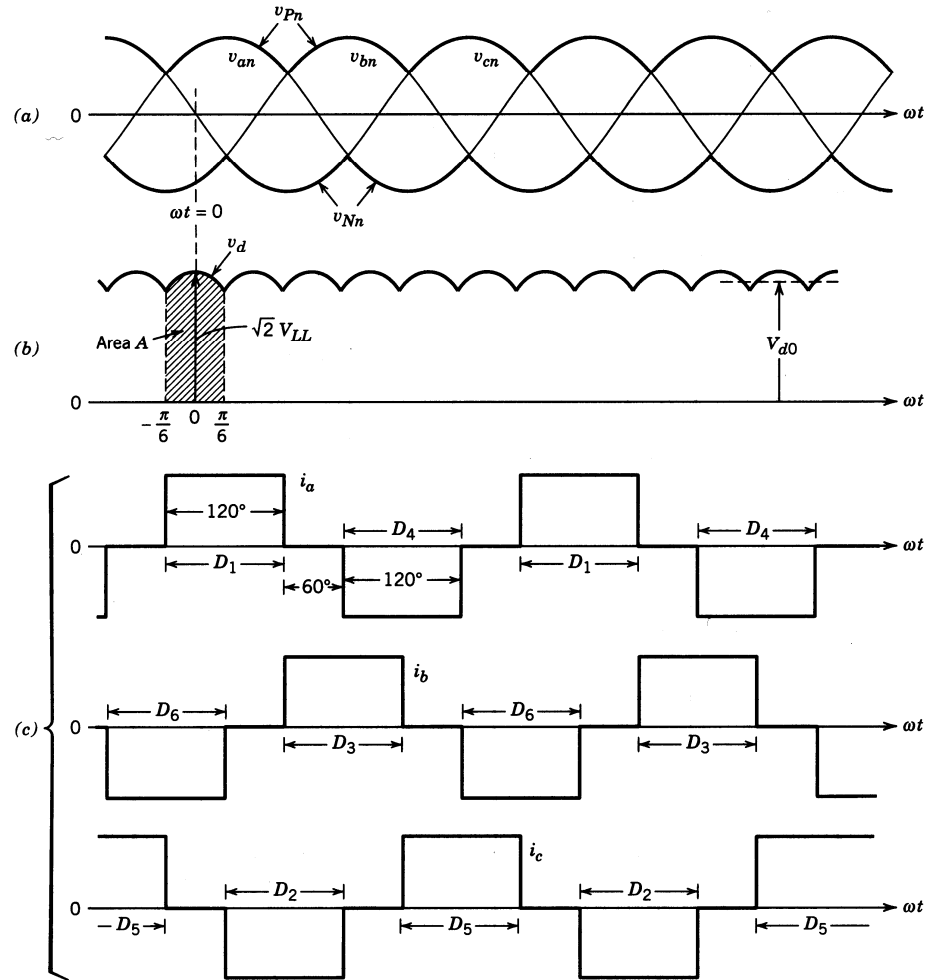


Figure 5-32 Waveforms in the circuit of Fig. 5-31.

6-Pulse Rectifier: Line Current

- Assuming output current to be purely dc and zero ac-side inductance

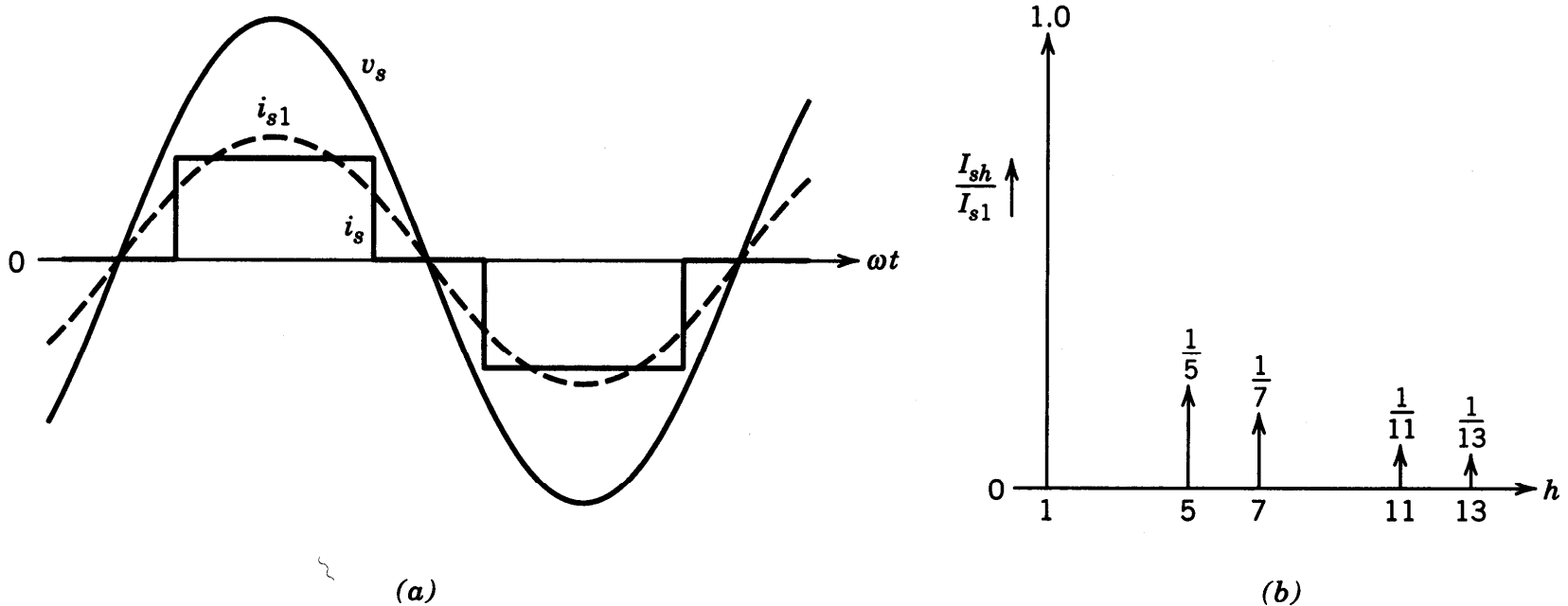
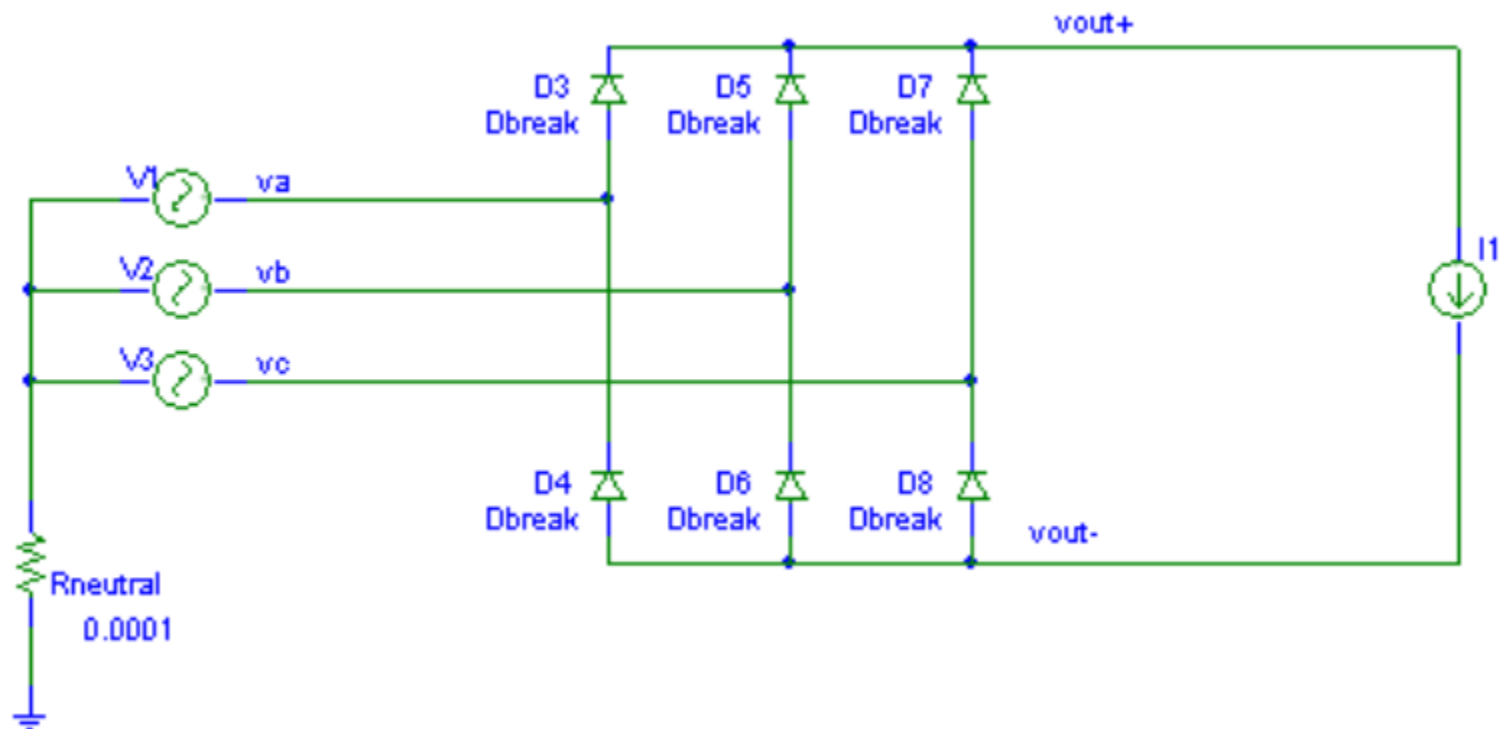


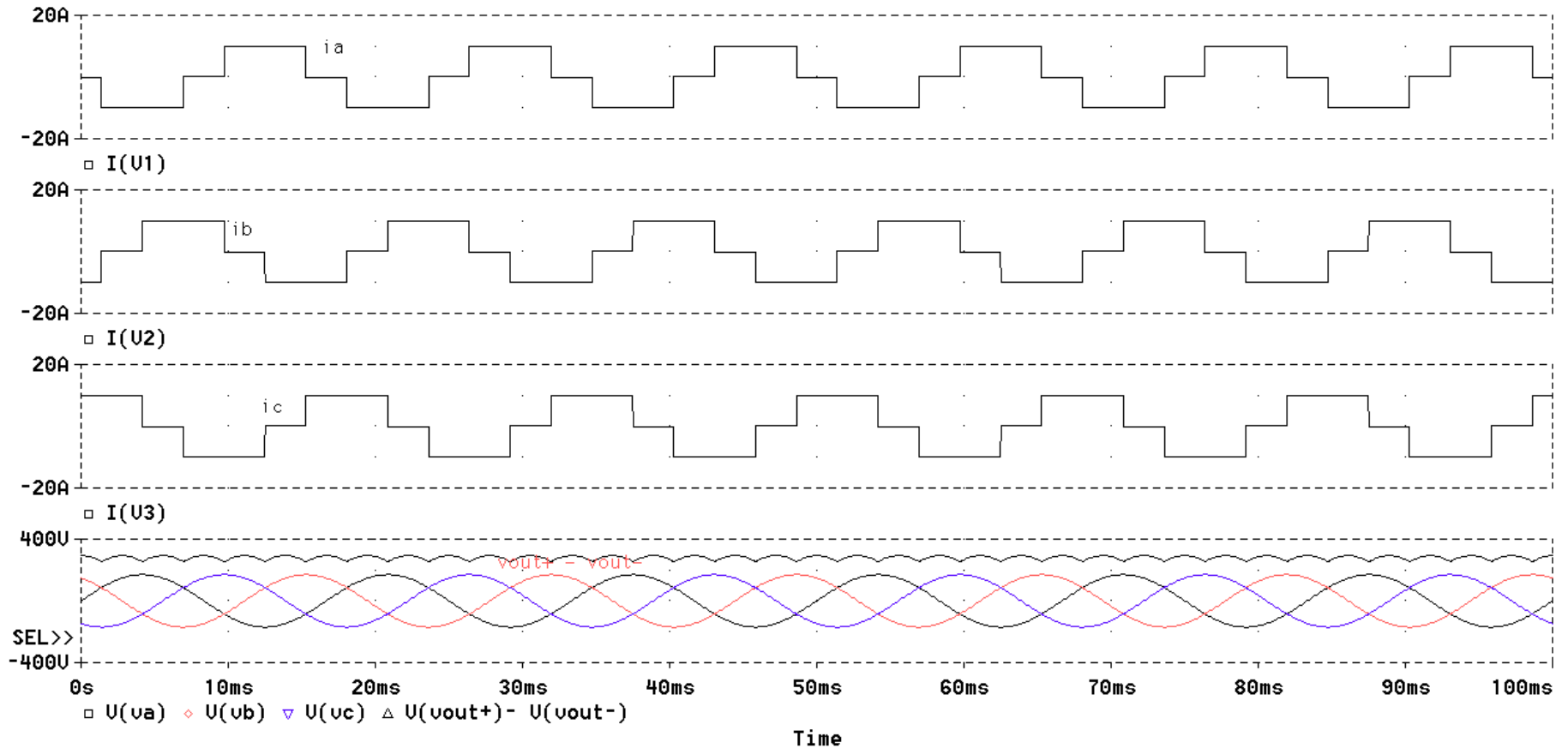
Figure 5-33 Line current in a three-phase rectifier in the idealized case with $L_s = 0$ and a constant dc current.

6-Pulse Rectifier with Current Source Load Simulation

- Simplified with $L_s = 0$ (no line inductance); and current source load

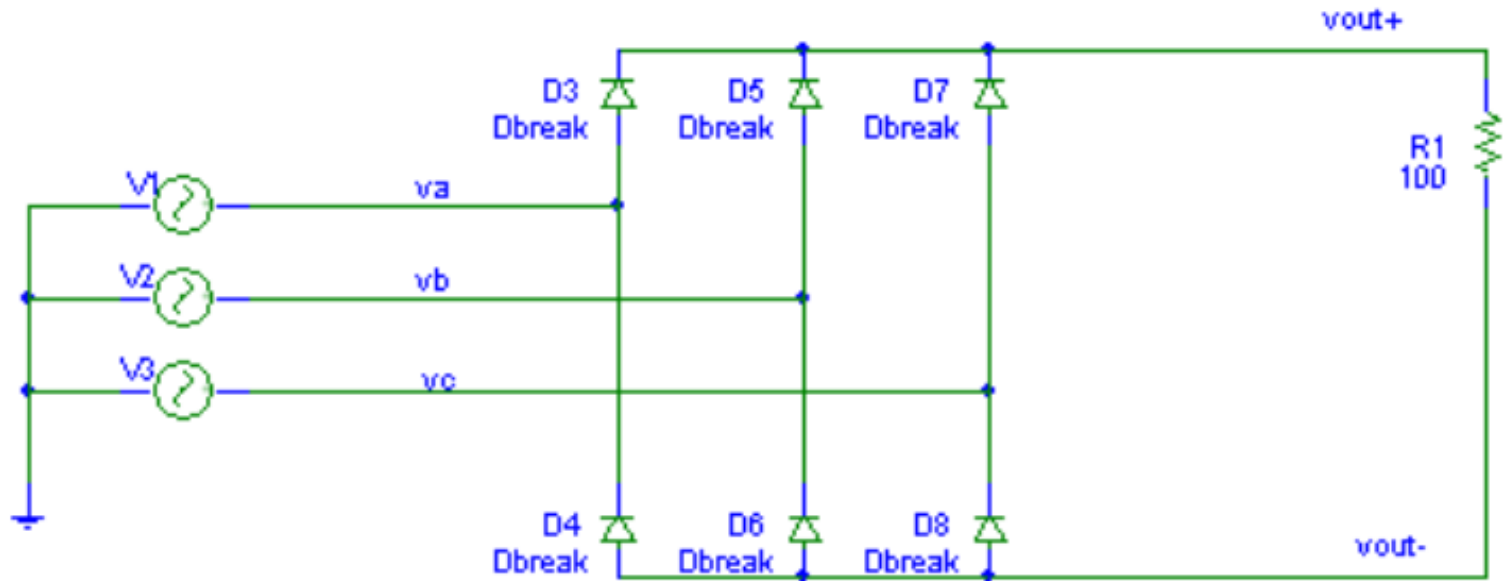


6-Pulse Rectifier with Current Source Load



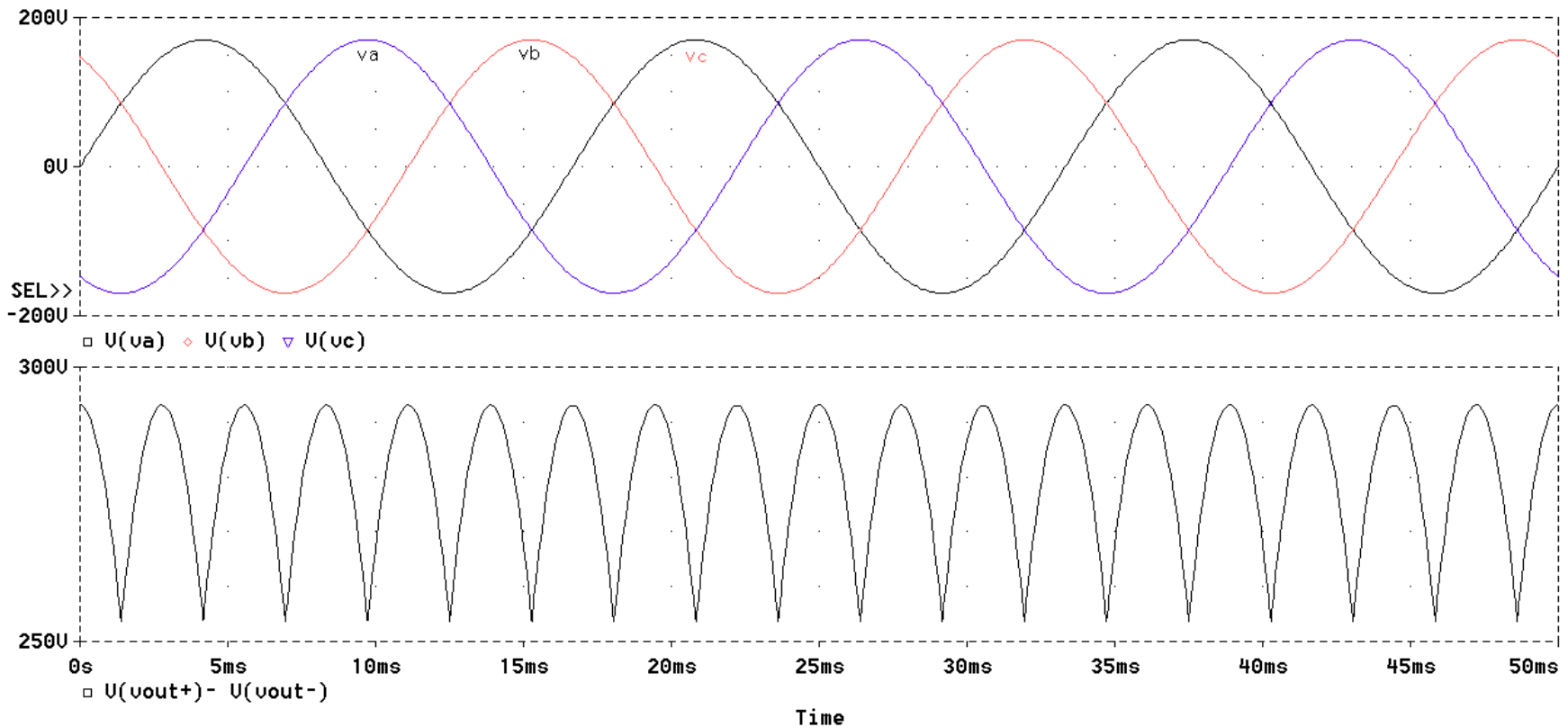
6-Pulse Rectifier with Resistive Load Simulation

- Simplified with $L_s = 0$ (no line inductance)



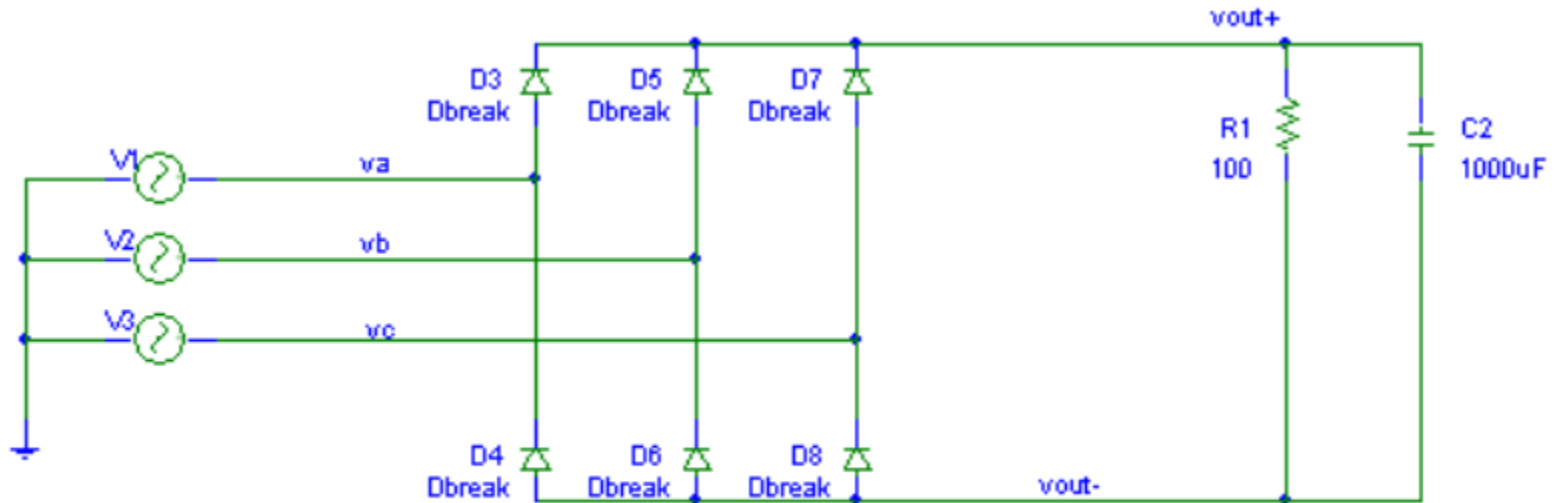
6-Pulse Rectifier with Resistive Load --- Output

- Note that fundamental of ripple frequency = 360 Hz
- Note that average output voltage is higher than single line voltage. Peak value is $\sqrt{3}$ x peak of line = 294V

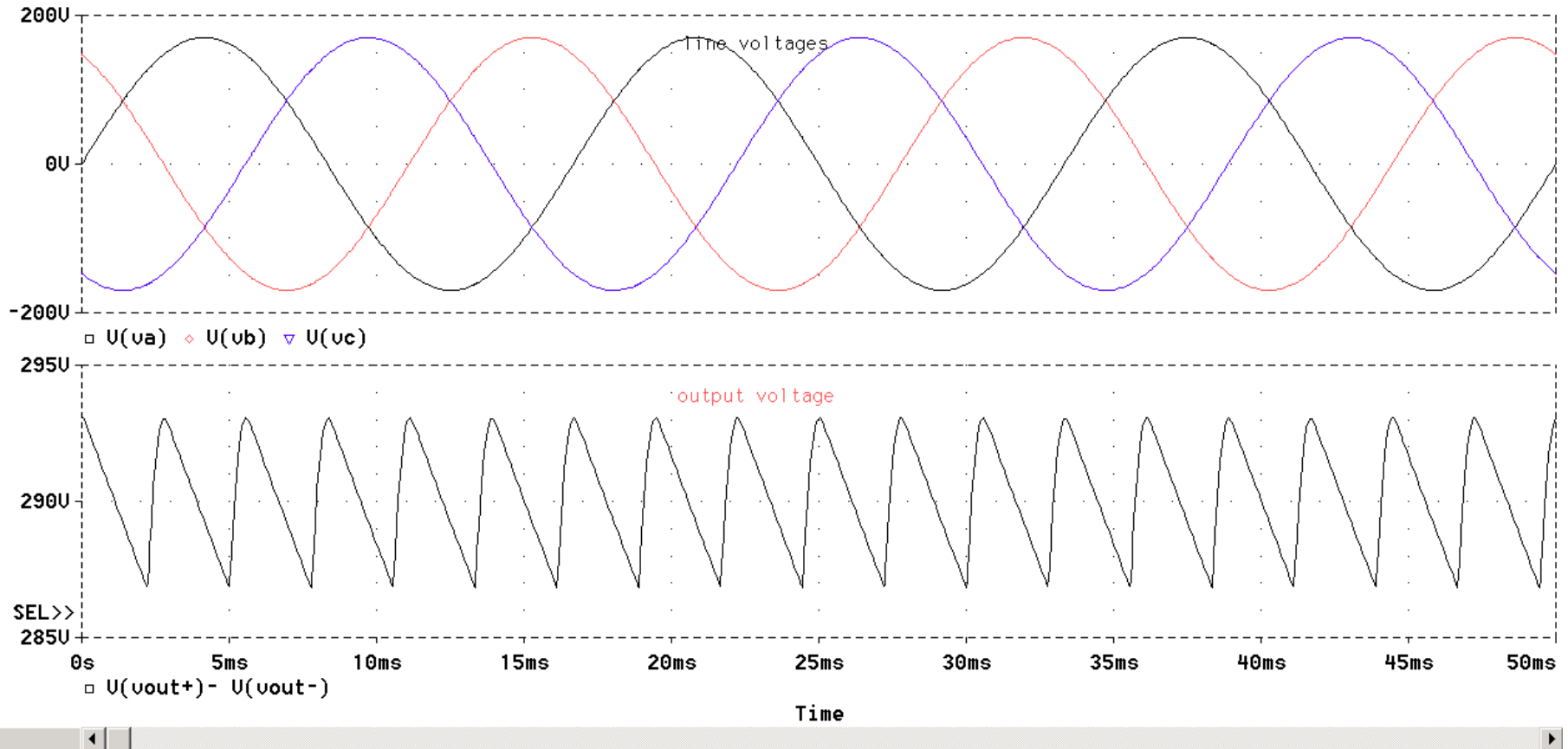


6-Pulse Rectifier with Resistive Load and Capacitor Filter

- Let's add filter capacitor
- Note that a smaller capacitor can be used for the 3 phase rectifier compared to single phase rectifier, because (1) Ripple is smaller and (2) Ripple frequency is higher

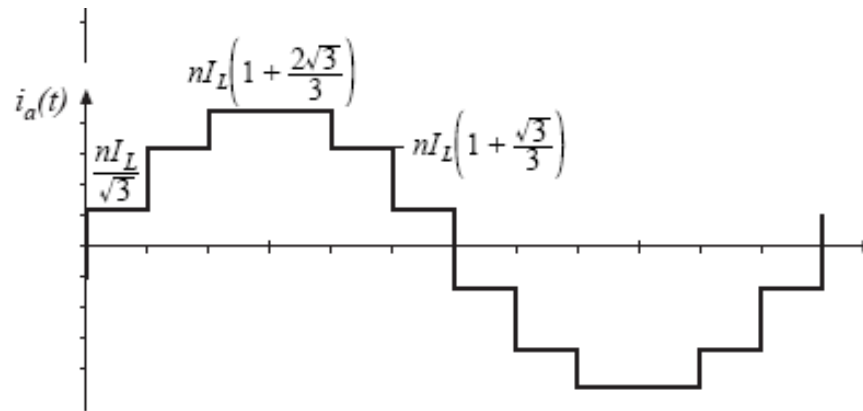
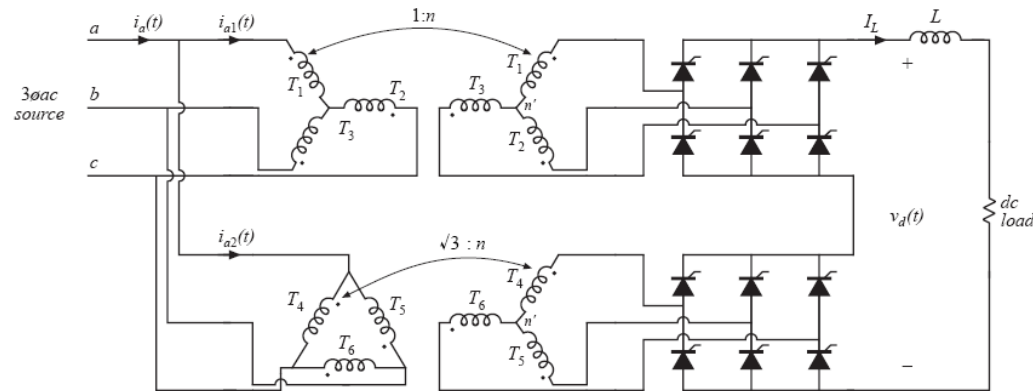


6-Pulse Rectifier with Resistive Load and Capacitor Filter



12-Pulse Rectifier

- Add 2 phase-shifted 6-pulse rectifiers; 5th and 7th are eliminated
- Only harmonics are the 11th, 13th, 23rd, 25th ...



Reference: R. W. Erickson and D. Maksimovic, Fundamentals of Power Electronics, 2d edition
Electromechanics

Three-Phase, Four-Wire System with Nonlinear Load

- With single-phase nonlinear load, there can be a neutral current

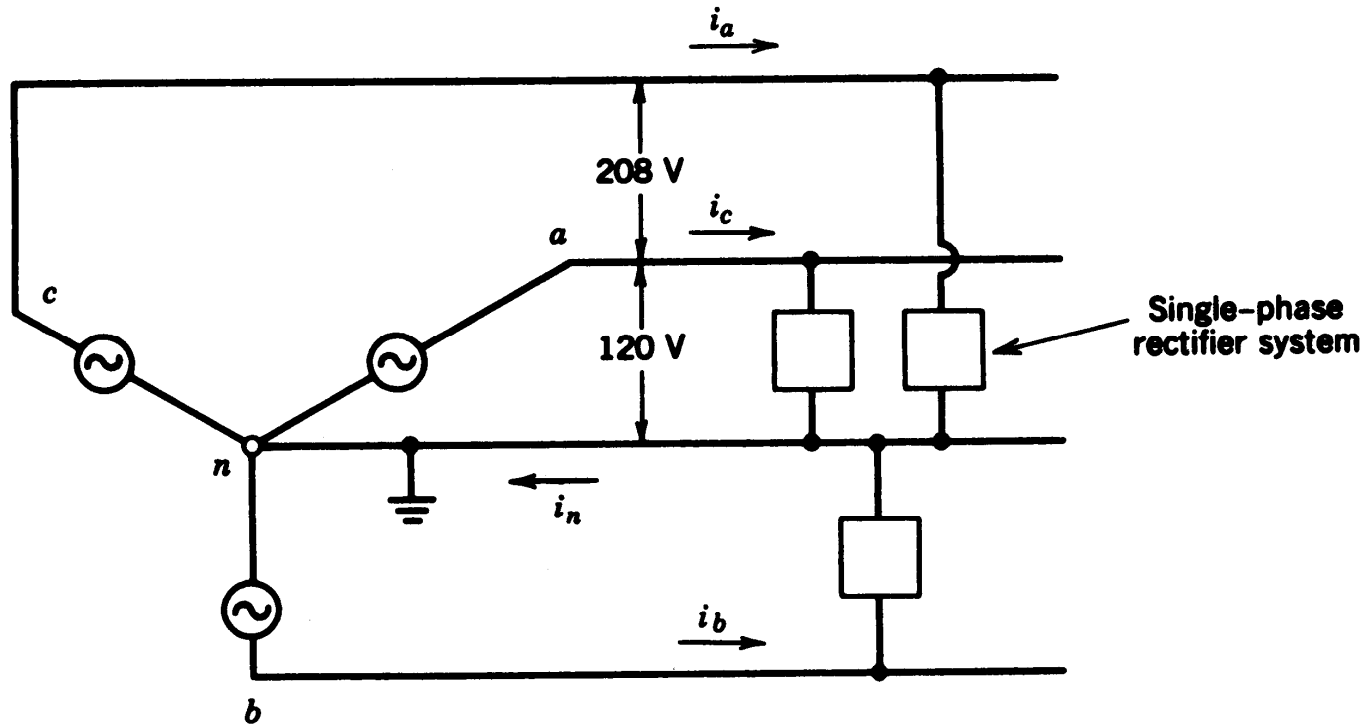


Figure 5-28 Three-phase, four-wire system.

Reference: Mohan, Undeland and Robbins, *Power Electronics, Converters, Applications and Design*, John Wiley, 2003, pp. 101

Neutral Current in a 3-Phase, Four-Wire System

- The neutral current can be very high if driving nonlinear loads line to neutral which generate 3rd or other harmonics
- If line currents are highly discontinuous, the neutral current can be as large as 1.73xline current 3rd harmonic
- Note 3rd harmonic here

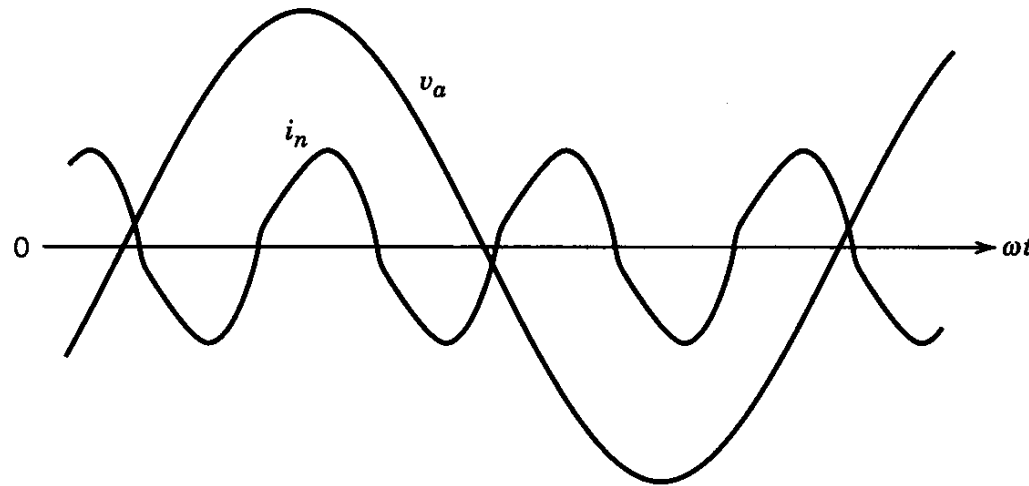
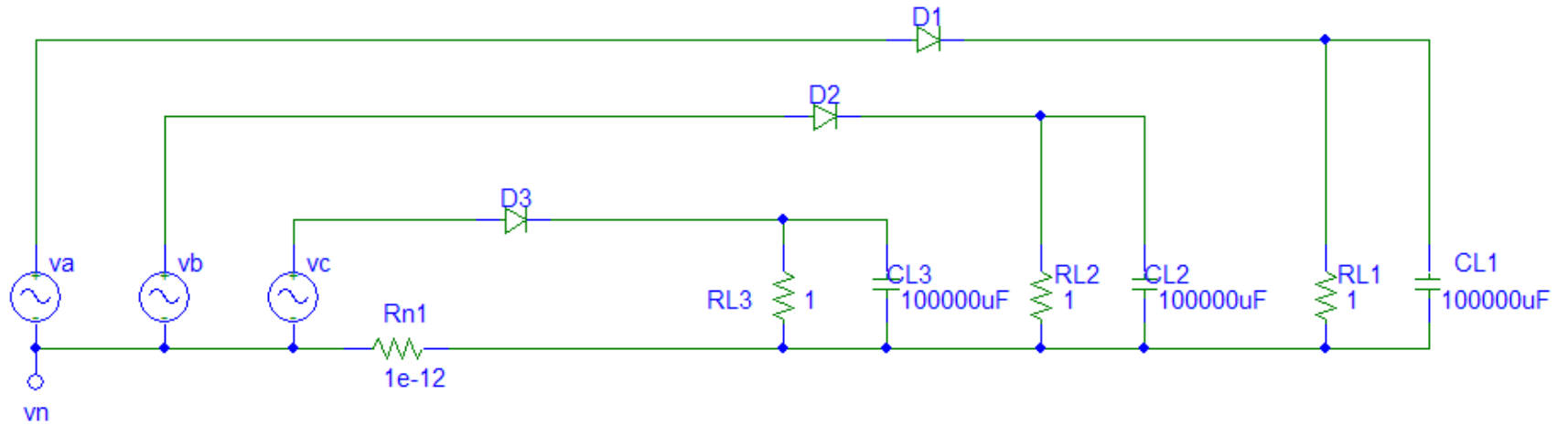


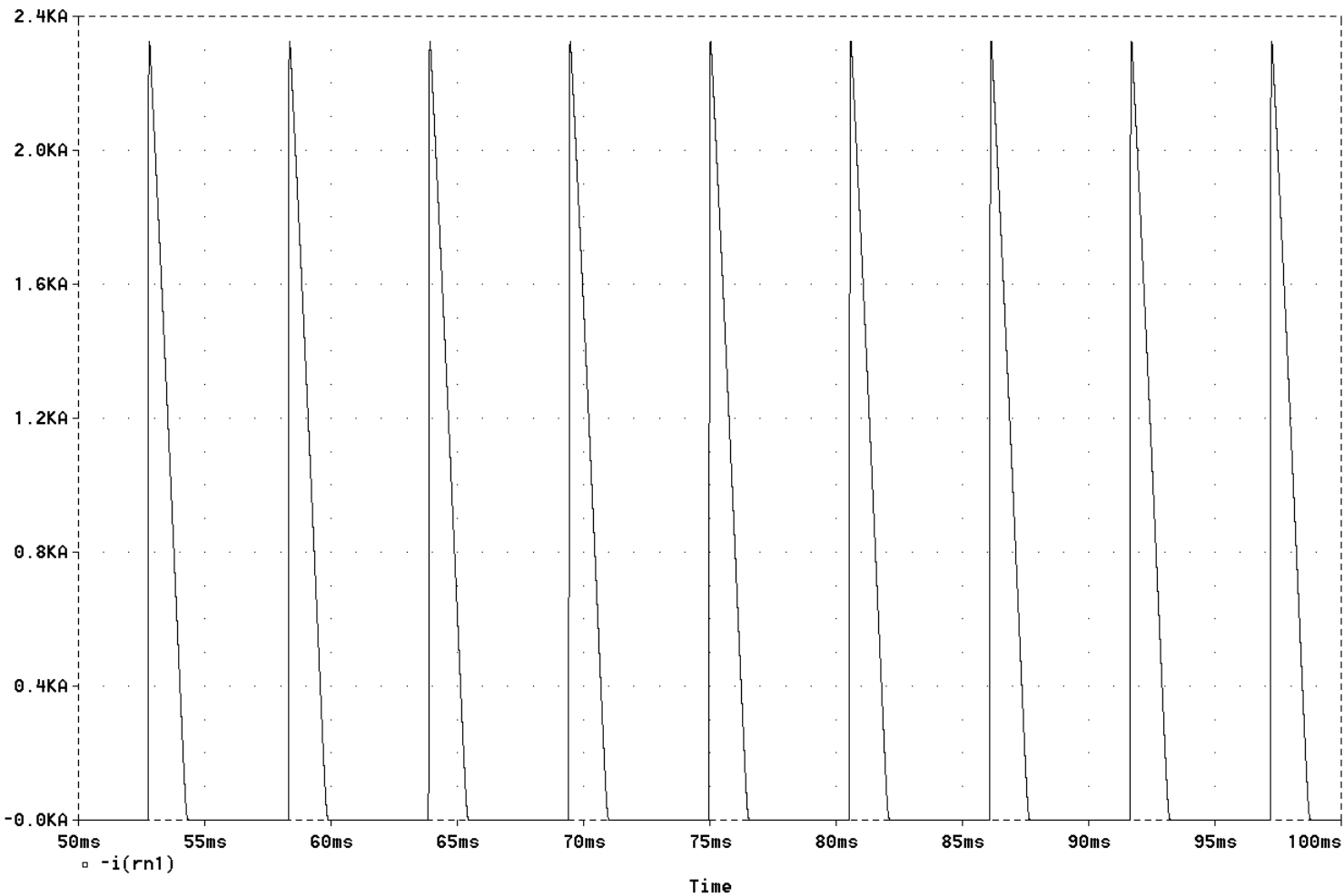
Figure 5-29 Neutral-wire current i_n .

Reference: Mohan, Undeland and Robbins, *Power Electronics, Converters, Applications and Design*, John Wiley, 2003, pp. 102

Simulation of Simple Case

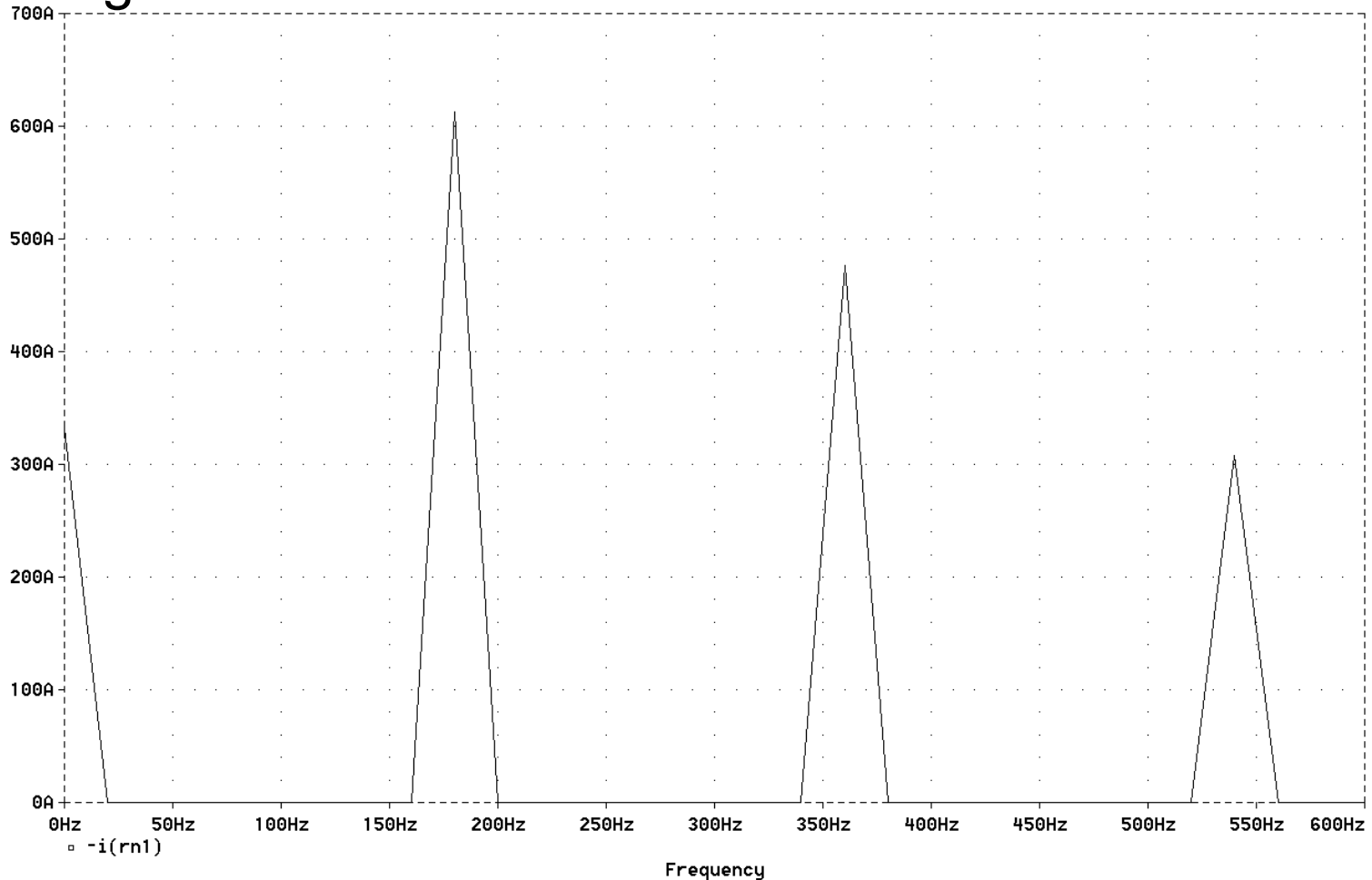


Simulation of Simple Case --- Neutral Current vs. Time



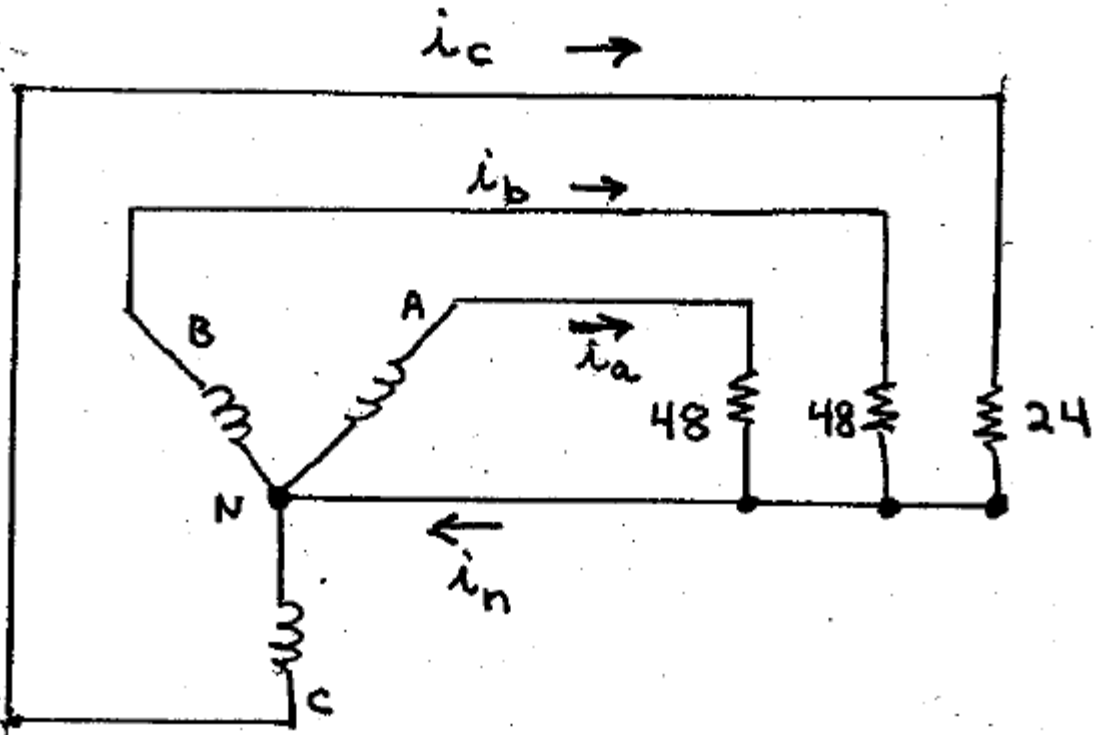
Simulation of Simple Case --- Spectrum of Neutral Current

- Note high 3rd harmonic



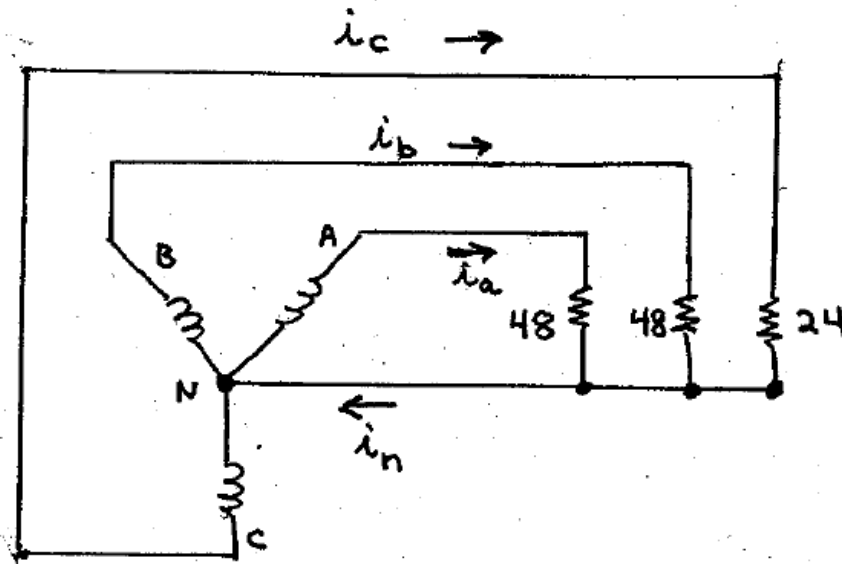
Example: Three-Phase Line Current Calculation

- Assume 60 Hz, 3-phase 4-wire, 480 V phase to phase (277V phase-neutral). Loads on phases A and B are 48 Ohms to neutral. Load on phase C is 24 Ohms to neutral. We'll find phase currents and neutral current



Example: Three-Phase Line Current Calculation

- Let's find phase currents:



$$v_{an} = \frac{480}{\sqrt{3}} \angle 0^\circ = 277 \angle 0^\circ$$

$$v_{bn} = \frac{480}{\sqrt{3}} \angle -120^\circ = 277 \angle -120^\circ$$

$$v_{cn} = \frac{480}{\sqrt{3}} \angle -240^\circ = 277 \angle -240^\circ$$

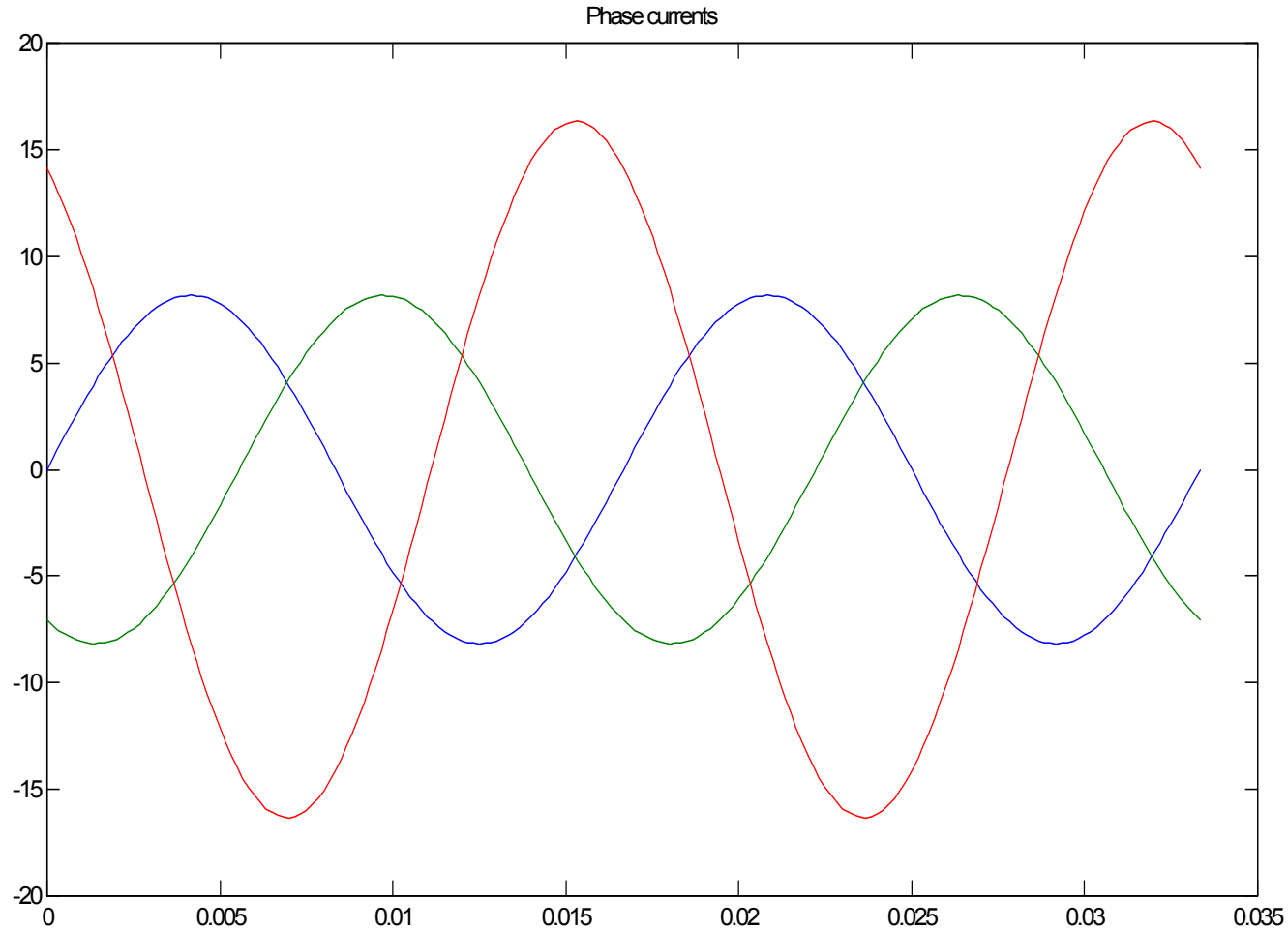
$$i_a = \frac{v_{an}}{48\Omega} = 5.77 \text{ A} \angle 0^\circ$$

$$i_b = \frac{v_{bn}}{48\Omega} = 5.77 \text{ A} \angle -120^\circ$$

$$i_c = \frac{v_{cn}}{24\Omega} = 11.54 \text{ A} \angle -240^\circ$$

Example: Three-Phase Line Current Calculation

- Phase currents



Example: Neutral Current Calculation

- Calculation of neutral current: remember that in a balanced, 3-phase system with linear loads the neutral current sums to zero.
- In this case, we have an imbalance because the amplitude of phase C current is higher than the others.
- The “leftover” current equals the neutral current

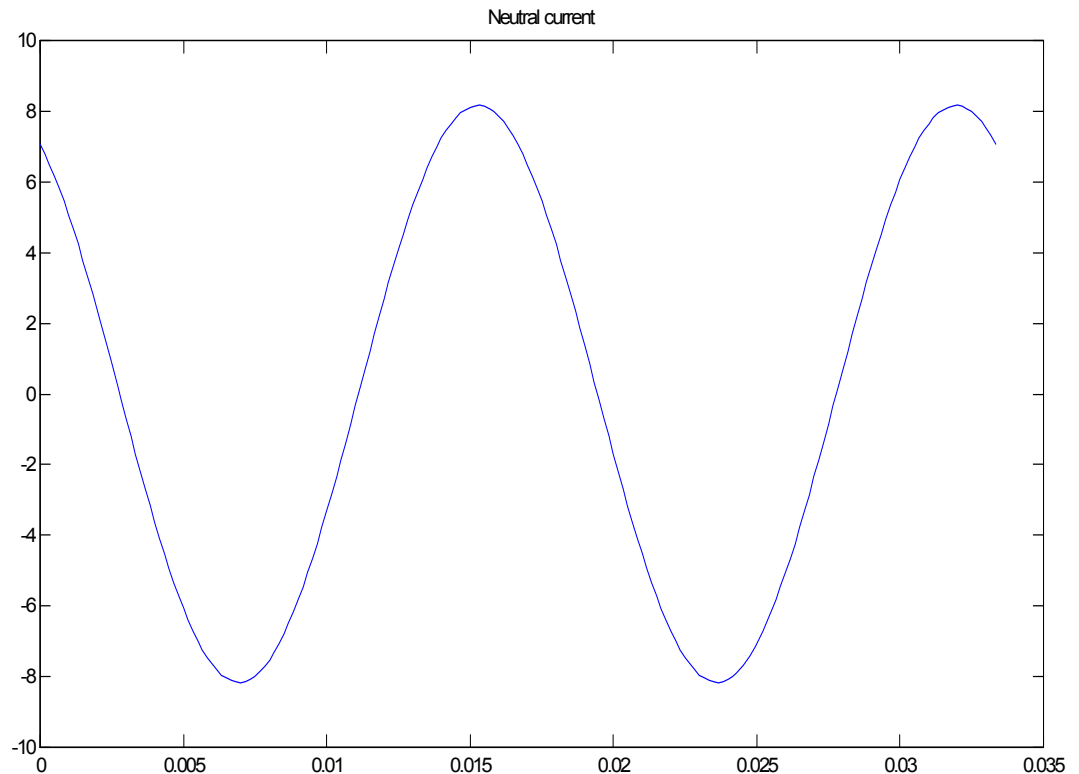
$$\begin{aligned}i_n &= i_a + i_b + i_c \\&= 5.77 \angle 0^\circ + 5.77 \angle -120^\circ + (5.77 \angle -240^\circ + 5.77 \angle -240^\circ) \\&= 5.77 \angle -240^\circ\end{aligned}$$

Example: Neutral Current Calculation

$$i_n = i_a + i_b + i_c$$

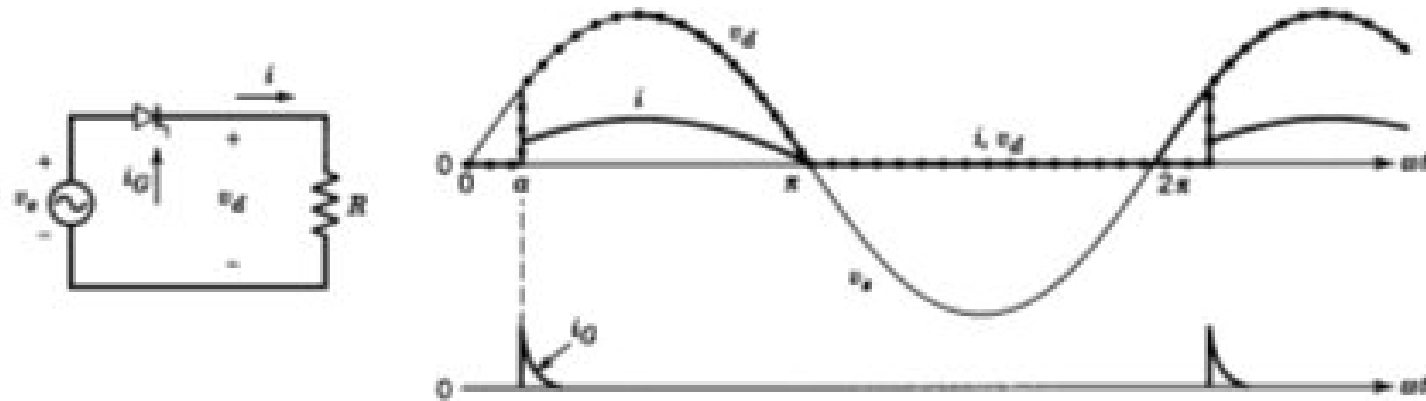
$$= 5.77 \angle 0^\circ + 5.77 \angle -120^\circ + (5.77 \angle -240^\circ + 5.77 \angle -240^\circ)$$

$$= 5.77 \angle -240^\circ$$



Thyristor Converter --- Single Phase with Resistive Load

- Angle α is called “firing angle”

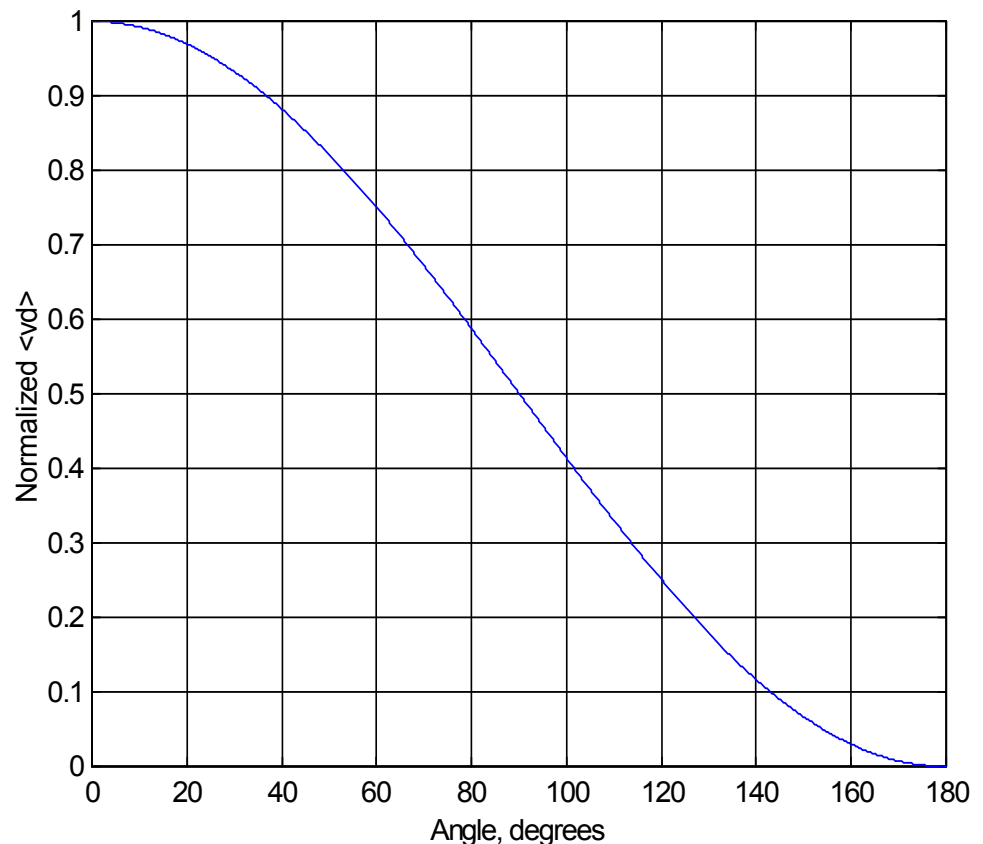


$$\langle v_d \rangle = \frac{V_{pk}}{2\pi} (1 + \cos \alpha)$$

Thyristor Converter --- Single Phase with Resistive Load

- Control characteristic: average output voltage vs. firing angle

$$\langle v_d \rangle = \frac{V_{pk}}{2\pi} (1 + \cos \alpha)$$



Three-Phase Thyristor Converter

- AC-side inductance is included

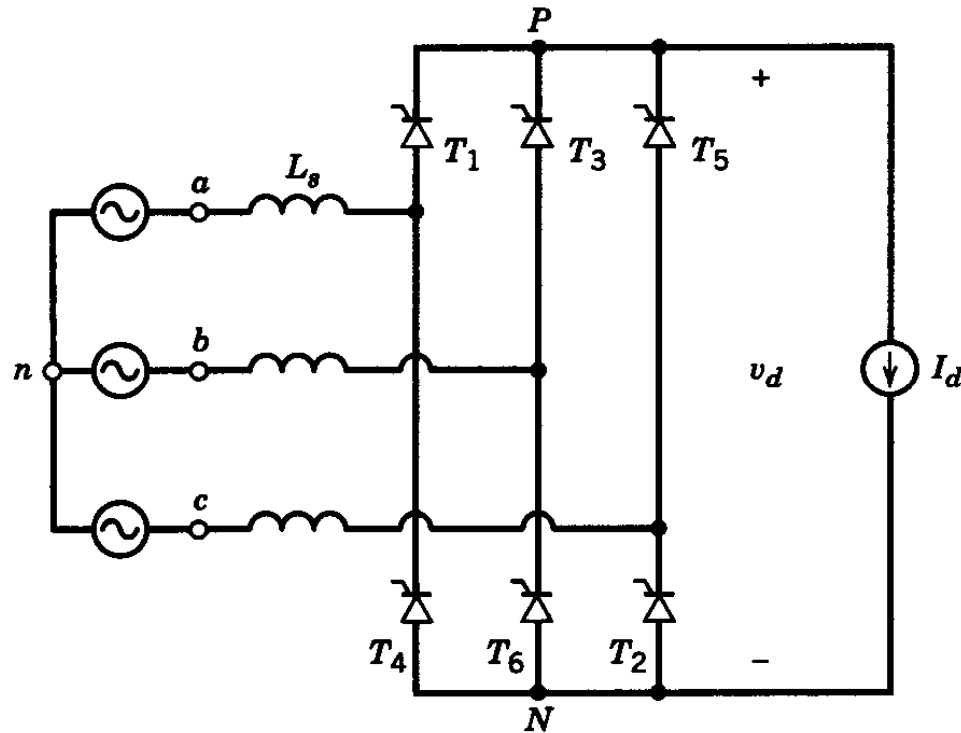


Figure 6-24 Three-phase converter with L_s and a constant dc current.

Today's Summary

- Today we've covered:
 - Signal basics, step response, resonance, etc.
 - Power in its various forms
 - Root-mean square (RMS)
 - Power cables and cable impedance
 - Power factor and power factor correction (PFC)
 - Fourier series and harmonics
 - Basic three-phase circuits