SCTC Analysis Estimates Low-Frequency –3-dB Point

Use Short-Circuit Time Constants To Guide The Choice Of Bypass- And Coupling-Capacitor Values In Linear-Circuit Design.

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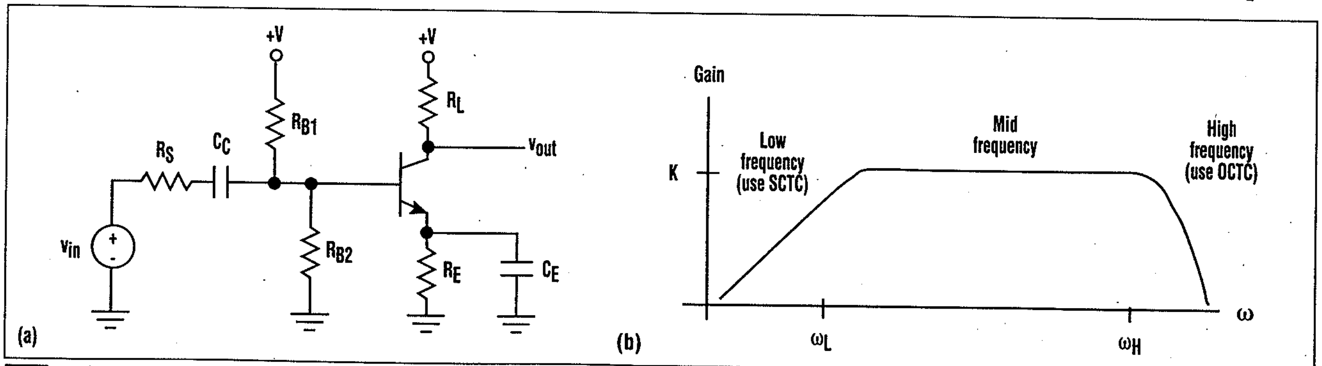
Open-circuit time constants is a method for approximating the high-frequency –3-dB point (ω_H) that was detailed in a previous article (ELECTRONIC DESIGN SPECIAL ANALOG ISSUE, *June 24, 1993, p. 41*). An analogous short-circuit time constants (SCTC) procedure allows designers to approximately calculate the low frequency –3-dB point (ω_L) of a linear circuit using the circuit's low-frequency incremental model.

The complete model for a typical ac-coupled transistor amplifier has four capacitors in it, two of which are due to internal transistor capacitances *(Fig. 1a)*. In many instances, the transistor amplifier has a wide mid-frequency range where gain is constant *(Fig. 1b)*. Therefore, the amplifier's full incremental model can be split into three simpler models—low frequency, mid-band, and high frequency.

In the low-frequency amplifier model, the effects of bypass and coupling capacitors dominate, and internal transistor capacitances behave as open-circuits. The mid-band model can ignore the effects of all circuit capacitances. In the high-frequency amplifier model, internal transistor capacitances dominate, and all bypass and coupling capacitors behave as short-circuits.

It's simple to determine the form of the transfer function for the amplifier



1. A TYPICAL ac-coupled transistor amplifier (a) has a wide mid-frequency range in which gain is constant (b). Consequently, the full incremental model can be split into three simpler models.

in Figure 1a if only low- and mid-frequency behavior is considered. In addition, the analysis won't account for high-frequency behavior due to internal transistor capacitances. The circuit has two capacitors, so there are two independent poles in the transfer function. Also, there's a zero at zero frequency due to the coupling capacitor C_C. Finally, there's a zero at higher frequency due to the emitter bypass capacitor C_E. A transfer function that meets these requirements is:

$$\frac{v_{out}}{v_{in}} = \frac{Ks(\tau_z s+1)}{(\tau_1 s+1)(\tau_2 s+1)} \quad (1)$$

This transfer function results in the gain-versus-frequency plot that's shown in Figure 1b for low and mid frequencies. Although it won't be proven here, all zeros of the transfer function are at lower frequencies than the highest pole. Therefore, the transfer function may be approximated by assuming that the zeros are at zero frequency:

$$\frac{v_{out}}{v_{in}} \approx \frac{Ks^2}{(\tau_1 s+1)(\tau_2 s+1)} \quad (2)$$

Multiplying out the denominator results in:

$$\frac{v_{out}}{v_{in}} \approx \frac{Ks^2}{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1} \quad (3)$$

Generally, each coupling and bypass capacitor has a relatively large value. This results in each time constant τ being relatively big. Therefore, throw out the +1 term in the denominator of the equation to make the following approximation valid at low frequencies:

$$\frac{v_{out}}{v_{in}} \approx \frac{Ks}{\tau_1\tau_2 s+(\tau_1+\tau_2)} \quad (4)$$

Using standard Bode-plot methods, it is clear that the –3-dB point of this transfer function occurs when:

$$\omega_L \approx \frac{\tau_1+\tau_2}{\tau_1\tau_2} \approx \frac{1}{\tau_1}+\frac{1}{\tau_2} \quad (5)$$

Equation 5 approximates the low-frequency breakpoint as the summation of two easily calculated frequencies. Following a line of reasoning similar to that used in open-circuit time constants analysis, the low-frequency breakpoint of a network is approximated by:

$$\omega_L \approx \sum_j \frac{1}{\tau_j} \quad (6)$$

where each τ_j is an RC time constant of every individual bypass or coupling capacitor with all other bypass and decoupling capacitors in the circuit shorted. Even though this example works through the math for a circuit with two capacitors, the method may be with equal validity to networks with more bypass and coupling capacitors.

DESIGN EXAMPLE

Consider the design of a single-transistor amplifier *(Fig. 2)*. The circuit uses a 2N3904 transistor, and has the following design specifications: a source resistance R_S of 1 kΩ, gain of greater than 100, and a low-frequency –3-dB point f_L of less than 1 kHz.

The SCTC analysis considers only the low-frequency behavior of the amplifier. Bias-point and load-resistor values have been chosen to give a mid-band gain of approximately –100. SCTC will help select reasonable values for the capacitors C_C and C_E based on the specification of low-frequency –3-dB point.

The transistor has the following parameters: collector current, I_C is 2 mA; r_x is 160 Ω; r_π is 2.6 kΩ; g_m is 0.08 mho; and h_{fe} is 200 *(Fig. 3a)*. The two short-circuit time constants for C_C and C_E are to be called τ_1 and τ_2. SCTC show that to meet the bandwidth specification, the following must hold:

$$\frac{1}{\tau_1}+\frac{1}{\tau_2} < 2\pi \times 1000 \quad (7)$$

To find τ_1, the short-circuit time constant for C_C, first turn off the input source v_{in} by shorting it. Then, short circuit C_E and find the resistance R_{1s} across the C_C terminals *(Fig. 3b)*. The time constant $\tau_1 = R_{1s}C_C$. Because a brief inspection reveals that the $g_m v_\pi$ generator has no effect on R_{1s}, the resistance is:

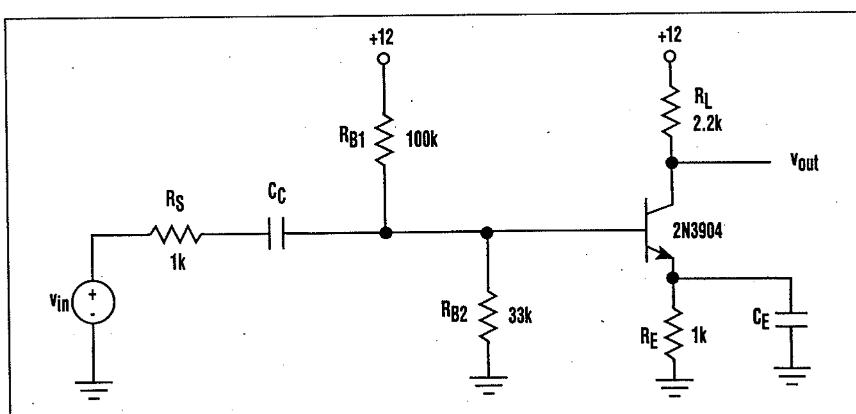
$$R_{1s} = R_s + (R_{B1}\|R_{B2})\|(r_x + r_\pi) \approx 3.5\ k\Omega \quad (8)$$

Find τ_2, the short-circuit time constant for C_E, by short circuiting C_C and calculating the resistance R_{2s} across the C_E terminals *(Fig. 3c)*. For the sake of simplicity, ignore the R_{B1} and R_{B2} resistances, because they are much larger than R_S. The resistance R_{2s} is found by applying a test-voltage source at the emitter terminal of the transistor and measuring the resulting current:

$$I_T = \frac{V_T}{R_E} + \frac{V_T}{R_s + r_x + r_\pi} + V_T\frac{g_m r_\pi}{R_s + r_x + r_\pi} \quad (9)$$

The first term in Equation 9 is current through the emitter resistor due to the voltage source; the second term is current through R_S, r_x, and r_π due to the voltage source alone; and the third term is current due to the $g_m v_\pi$ generator. Recognizing that $g_m r_\pi = h_{fe}$, the resistance facing C_E is given by:

$$R_{2s} = \frac{V_T}{I_T} = R_E \left\| \frac{R_s + r_x + r_\pi}{1 + h_{fe}} \right. \approx 18\ \Omega \quad (10)$$



2. THIS AMPLIFIER is built with just one 2N3904 transistor, and has gain of greater than 100 for ω_L of less that 1 kHz.

Now, the final choice of capacitor values is constrained by Equation 7, which sets the minimum values of time constants τ_1 and τ_2. Arbitrarily "divide up" the time constants equally between the two capacitors.

$$\frac{1}{\tau_1}=\frac{1}{R_{1s}C_C}<2\pi\times 500 \quad (11)$$

$$\frac{1}{\tau_2}=\frac{1}{R_{2s}C_E}<2\pi\times 500 \quad (12)$$

This gives design values:

$$C_C>\frac{1}{2\pi\times 500\times R_{1s}}>0.09\ \mu F \quad (13)$$

$$C_E>\frac{1}{2\pi\times 500\times R_{2s}}>18\ \mu F \quad (14)$$

Chosen off-the-shelf values are C_C = 0.1 μF, and C_E = 20 μF.

The method of short-circuit time constants shows that a relatively small value of coupling capacitor C_C will suffice due to the large resistance seen at its terminals. It also makes sense that C_E must be a large capacitor value, because the output resistance of an emitter follower is very small.

CAVEAT EMPTOR

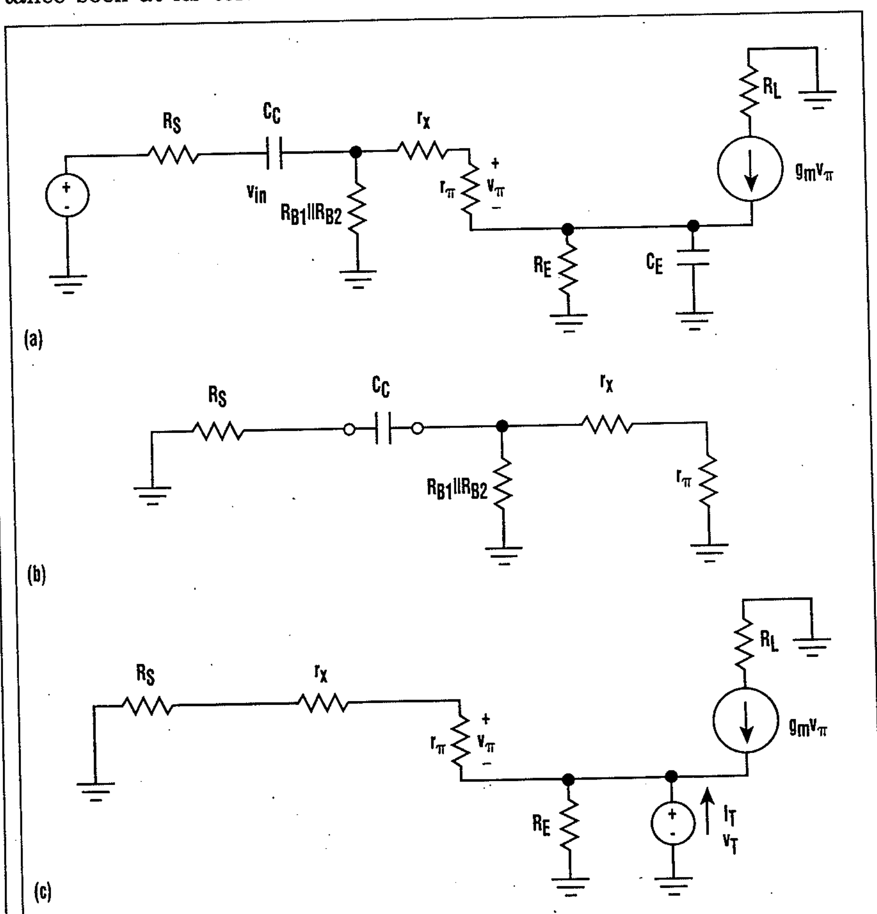
It's obvious by inspecting Figure 2 that capacitors C_C and C_E affect the low- and mid-frequency behavior of the circuit. For instance, if capacitor C_C were open-circuited, the mid-band gain of the amplifier would be zero. If capacitor C_E were open-circuited, the mid-band gain of the amplifier would be about –2.

There are cases, however, where bypass capacitors have little or no effect on the mid-frequency performance of an amplifier. For instance, in a common-emitter amplifier with a cascode, the resistive network biasing the base of the cascode transistor is usually bypassed with a capacitor. The bypassing ensures better high-frequency performance, but has little effect at low and mid frequencies. This capacitor should not be included in the short-circuit time constants analysis. So, the moral is: think carefully about how the circuit operates before applying the short-circuit time constants method to a given capacitor in a circuit.

RESULTS

A Spice run shows that the gain of the circuit is approximately –110, with a low-frequency –3-dB point of 500 Hz. (Note that Spice was not used for design, only for verification). The answer obtained from using SCTC was not exact, but who cares? It's more important that engineers develop design insight by using such methods.

Short-circuit time constants offer guidance for choosing bypass- and coupling-capacitor values. Using this method, a design may be optimized for preferred capacitor values based on such criteria as size or cost. For instance, this article's design example divided up the time constant equally between the two capacitors, but any other combination will also work. As with the method of open-circuit time constants, all results are approximate. But the value of such methods is the design insight the method provides to the designers, not exact answers.☐



3. THE LOW-FREQUENCY model is used to calculate the short-circuit time constants (a). Shorting C_C and C_E helps determine the time constants τ_1 and τ_2, respectively (b,c).

References:

Gray, Paul E., and Searle, Campbell L., *Electronic Principles Physics, Models, and Circuits.* John Wiley, 1967.

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